

Voting Methods

Preference ballots are ballots in which a voter is asked to rank all the candidates in order of preference.

A **preference table** shows how often each particular outcome occurred.

There were $14 + 10 + 8 + 4 + 1$, or 37 voters in the election.

Number of Votes	14	10	8	4	1
First Choice	P	S	T	R	S
Second Choice	R	R	S	T	T
Third Choice	S	T	R	S	R
Fourth Choice	T	P	P	P	P

14 ballots rank the candidates P, R, S, T.

10 ballots rank the candidates S, R, T, P.

8 ballots rank the candidates T, S, R, P.

4 ballots rank the candidates R, T, S, P.

1 ballot ranks the candidates S, T, R, P.

How many people selected Donna (D) as their first choice?

Number of Votes	130	120	100	150
First Choice	A	D	D	C
Second Choice	B	B	B	B
Third Choice	C	C	A	A
Fourth Choice	D	A	C	D

Solution: We find the number of people who voted for D as their first choice by reading across the row that says *First Choice*. When you see a D in this row, write the number above it. Then find the sum of the numbers: $120 + 100 = 220$. Thus, 220 people selected Donna as their first choice.

The Plurality Method:

The candidate (or candidates, if there is more than one) with the most first-place votes is the winner.

Example: For the previous preference table, who is declared the winner using the plurality method? Solution: The candidate with the most first-place votes is the winner. When using the preference table, we only need to look at the row indicating the number of first-choice votes.

P received 14 first-place votes.
S received $10 + 1 = 11$ first-place votes.
T received 8 first-place votes.
R received 4 first-place votes.

Number of Votes	14	10	8	4	1
First Choice	P	S	T	R	S
Second Choice	R	R	S	T	T
Third Choice	S	T	R	S	R
Fourth Choice	T	P	P	P	P

The Borda Count Method:

Each voter ranks the candidates from the most favorable to the least favorable.

1. Each last-place vote is given 1 point, each next-to-last-place vote is given 2 points, each third-from-last-place vote is given 3 points, and so on.
2. The points are totaled for each candidate separately.
3. The candidate with the most points is the winner.

Example: In the previous table, who is the winner using the **Borda** method? **Solution:** Because there are four candidates, a first-place vote is worth 4 points, a second place vote is worth 3 points, a third-place vote is worth 2 points, and a fourth-place vote is worth 1 point.

Number of Votes	14	10	8	4	1
First Choice: 4 pts	P: $14 \times 4 = 56$	S: $10 \times 4 = 40$	T: $8 \times 4 = 32$	R: $4 \times 4 = 16$	S: $1 \times 4 = 4$
Second Choice: 3 pts	R: $14 \times 3 = 42$	R: $10 \times 3 = 30$	S: $8 \times 3 = 24$	T: $4 \times 3 = 12$	T: $1 \times 3 = 3$
Third Choice: 2 pts	S: $14 \times 2 = 28$	T: $10 \times 2 = 20$	R: $8 \times 2 = 16$	S: $4 \times 2 = 8$	R: $1 \times 2 = 2$
Fourth Choice: 1 pt	T: $14 \times 1 = 14$	P: $10 \times 1 = 10$	P: $8 \times 1 = 8$	P: $4 \times 1 = 4$	P: $1 \times 1 = 1$

Now, we read down each column and total the points for each candidate separately:

P: $56 + 10 + 8 + 4 + 1 = 79$ points

R: $42 + 30 + 16 + 16 + 2 = 106$ points

S: $28 + 40 + 24 + 8 + 4 = 104$ points

T: $14 + 20 + 32 + 12 + 3 = 81$ points

**Because Rita (R) has received the most points, she is the winner and the new president of the Student Film Institute.

The Plurality-with-Elimination Method

The candidate with the majority of first-place votes is the winner.

If no candidate receives a majority of first-place votes, eliminate the candidate (candidates, if there is a tie) with the fewest first-place votes from the preference table.

Move the candidates in each column below each eliminated candidate up one place.

The candidate with the majority of first-place votes in the new preference table is the winner.

If no candidate receives a majority of first-place votes, repeat this process until a candidate receives a majority.

The Pairwise Comparison Method

Voters rank all the candidates and the results are summarized in a preference table.

The table is used to make a series of comparisons in which each candidate is compared to each of the other candidates.

For each pair of candidates, X and Y, use the table to determine how many voters prefer X to Y and vice versa.

If a majority prefer X to Y, then X receives 1 point.

If a majority prefers Y to X, then Y receives 1 point.

If the candidates tie, then each receives half a point.

After all comparisons have been made, the candidate receiving the most points is the winner.

Fairness criterion:

The Majority Criterion: If a candidate receives a majority of first-place votes in an election, then that candidate should win the election.

The Head-to-Head Criterion: If a candidate is favored when compared separately-that is, head-to-head, with every other candidate, then that candidate should win the election.

Monotonicity Criterion: If a candidate wins an election and, in a reelection, the only changes are changes that favor that candidate, then that candidate should win the reelection.

The Irrelevant Alternatives Criterion: If a candidate wins an election and, in a recount, the only changes are that one or more of the other candidates are removed from the ballot, then that candidate should still win the election.

The Search for a Fair Voting System Arrow's Impossibility Theorem:

It is mathematically impossible for any democratic voting system to satisfy each of the four fairness criteria.

In 1951, economist Kenneth Arrow proved that there does not exist, and will never exist, any democratic voting system that satisfies all of the fairness criteria.