Number Bases in Positional Systems

Changing to Base Ten

1. Find the place value for each digit in the numeral.
2. Multiply each digit in the numeral by its respective place value.
3. Find the sum of the products in step 2.

Example: Convert 4726_{eight} to base ten.

Solution: The given base eight numeral has four places. From left to right, the place values are

8^3, 8^2, 8^1, and 1

$$\begin{aligned}4726_{\text{eight}} &= (4\times8^3)+(7\times8^2)+(2\times8^1)+(6\times1)\\ &= (4\times8\times8\times8)+(7\times8\times8)+(2\times8)+(6\times1)\\ &= 2048+448+16+6\\ &= 2518\end{aligned}$$

Changing Base Ten Numerals to Numerals in Other Bases

One way to convert from base 10 to a different base is to:

1. Determine the highest power of the base that goes into the number a non-zero number of times.
2. Determine how many times this power can be subtracted from the number without the result being negative (i.e, divide the number by the power). Write this digit down.
3. Redefine the number to be this smallest positive remainder upon division by the power in question
4. Redefine the power to be the power divided by the base.
5. Go back to step 2, unless the power is now less than one -- in which case, you are done.

For example, to convert 1073 to base 5, we recall that:

$5^0 = 1$

$5^1 = 5$

$5^2 = 25$

$5^3 = 125$

$5^4 = 625$

$5^5 = 3125$

Then we notice that $5^4 = 625$ is the highest power of 5 under 1073.

1073 = **1** * 625 + 448

448 = **3** * 125 + 73

73 = **2** * 25 + 23

23 = **4** * 5 + 3

3 = **3** * 1 + 0

Therefore, **13243** is the base 5 representation of 1073.

Divisibility

If *a* and *b* are natural numbers, *a* is ***divisible*** by *b* if the operation of dividing *a* by *b* leaves a remainder of 0.

This is the same as saying that *b* is a ***divisor*** of *a*, or *b* divides *a*.

This is symbolized by writing $b|a$.

TABLE 5.1 Rules of Divisibility

Divisible By	Test	Example
2	The last digit is 0, 2, 4, 6, or 8.	5,892,796 is divisible by 2 because the last digit is 6.
3	The sum of the digits is divisible by 3.	52,341 is divisible by 3 because the sum of the digits is $5 + 2 + 3 + 4 + 1 = 15$, and 15 is divisible by 3.
4	The last two digits form a number divisible by 4.	3,947,136 is divisible by 4 because 36 is divisible by 4.
5	The number ends in 0 or 5.	28,160 and 72,805 end in 0 and 5, respectively. Both are divisible by 5.
6	The number is divisible by both 2 and 3. (In other words, the number is even and the sum of its digits is divisible by 3.)	954 is divisible by 2 because it ends in 4. 954 is also divisible by 3 because the digit sum is 18, which is divisible by 3. Because 954 is divisible by both 2 and 3, it is divisible by 6.
8	The last three digits form a number that is divisible by 8.	593,777,832 is divisible by 8 because 832 is divisible by 8.
9	The sum of the digits is divisible by 9.	5346 is divisible by 9 because the sum of the digits, 18, is divisible by 9.
10	The last digit is 0.	998,746,250 is divisible by 10 because the number ends in 0.
12	The number is divisible by both 3 and 4. (In other words, the sum of the digits is divisible by 3 and the last two digits form a number divisible by 4.)	614,608,176 is divisible by 3 because the digit sum is 39, which is divisible by 3. It is also divisible by 4 because the last two digits form 76, which is divisible by 4. Because 614,608,176 is divisible by both 3 and 4, it is divisible by 12.

The Integers. Order of Operations

The set consisting of the natural numbers, 0, and the negatives of the natural numbers is called the set of ***integers***.

Addition of Integers

Rules

If the integers have the same sign,

1. Add their absolute values.
2. The sign of the sum is the same sign of the two numbers.

If the integers have different signs,

1. Subtract the smaller absolute value from the larger absolute value.
2. The sign of the sum is the same as the sign of the number with the larger absolute value.

Subtraction of Integers

For all integers *a* and *b*,

$a - b = a + (-b)$.

In words, to subtract *b* from *a*, add the additive inverse of *b* to *a*. The result of subtraction is called the ***difference***.

Multiplication of Integers: Rules

1. The product of two integers with the same signs is found by multiplying their absolute values. The product is positive.
2. The product of two integers with different signs is found by multiplying their absolute values. The product is negative.

Exponential Notation

Because exponents indicate repeated multiplication, rules for multiplying can be used to evaluate exponential expressions.

Property	Meaning	Examples
The Product Rule $b^m \cdot b^n = b^{m+n}$	When multiplying exponential expressions with the same base, add the exponents. Use this sum as the exponent of the common base.	$9^6 \cdot 9^{12} = 9^{6+12}$ $= 9^{18}$
The Power Rule $(b^m)^n = b^{mn}$	When an exponential expression is raised to a power, multiply the exponents. Place the product of the exponents on the base and remove the parentheses.	$(3^4)^5 = 3^{4 \cdot 5} = 3^{20}$ $(5^3)^8 = 5^{3 \cdot 8} = 5^{24}$
The Quotient Rule	When dividing exponential expressions with the same base, subtract the exponent in the denominator from the exponent in the numerator. Use this difference as the exponent of the common base.	

If b is any real number other than 0, $b^0 = 1$.
If b is any real number other than 0 and m is a natural number,
$b^{-m} = \frac{1}{b^m}$.

Scientific Notation

A positive number is written in scientific notation when it is expressed in the form $a \times 10^n$

Write each number in decimal notation:

a. 1.375×10^{10}   **b. 1.1×10^{-4}**

Order of Operations

1. Perform all operations within grouping symbols.
2. Evaluate all exponential expressions.
3. Do all the multiplications and divisions in the order in which they occur, working from left to right.
4. Finally, do all additions and subtractions in the order in which they occur, working from left to right.

Example: Simplify $6^2 – 24 \div 2^2 \cdot 3 + 1$. Ans: 19.

Rational Numbers

The set of ***rational numbers*** is the set of all numbers which can be expressed in the form a/b where a and b are integers and b is not equal to 0.

The integer a is called the ***numerator***.

The integer b is called the ***denominator***.

Density of Rational Numbers

If r and t represent rational numbers, with $r < t$, then there is a rational number s such that s is between r and t:

$r < s < t$.

The Irrational Numbers

The set of ***irrational numbers*** is the set of numbers whose decimal representations are neither terminating nor repeating.

The principal square root of a nonnegative number n, written $\sqrt{n}$ is the positive number that when multiplied by itself gives n. Non- exact roots are irrational.