Algebra: Equations and Inequalities

Algebraic Expressions and Formulas

An algebraic expression is a combination of variables and numbers using the operations of addition, subtraction, multiplication, or division as well as powers or roots.

Evaluate $7 + 5(x - 4)^3$ for $x = 6$

Solution:

$7 + 5(x - 4)^3 = 7 + 5(6 - 4)^3$

$= 7 + 5(2)^3$

$= 7 + 5(8)$

$= 7 + 40$

$= 47$

An **equation** is formed when an equal sign is placed between two algebraic expressions.

The mathematical model $W = -66x^2 + 526x + 1030$ describes the number of calories needed per day by women in age group x with moderately active lifestyles.

Vocabulary of Algebraic Expressions

Term: Those parts of an algebraic expression separated by addition.

Example: in the expression $7x - 9y - 3$

Coefficient: The numerical part of a term.

$7, -9$

Constant: A term that consists of just a number, also called a **constant term.** -3

Like terms: Terms that have the exact same variable factors. $7x$ and $3x$

Factors: Parts of each term that are multiplied.

Properties of Real Numbers

Property	Example
Commutative Property of Addition $a + b = b + a$	$13x^2 + 7x = 7x + 13x^2$
Commutative Property of Multiplication $ab = ba$	$x \cdot 6 = 6x$
Associative Property of Addition $(a + b) + c = a + (b + c)$	$3 + (8 + x) = (3 + 8) + x = 11 + x$
Associative Property of Multiplication $(ab)c = a(bc)$	$-2(3x) = (-2 \cdot 3)x = -6x$
Distributive Property $a(b + c) = ab + ac$	$5(3x + 7) = 5 \cdot 3x + 5 \cdot 7 = 15x + 35$
$a(b - c) = ab - ac$	$4(2x - 5) = 4 \cdot 2x - 4 \cdot 5 = 8x - 20$

Example: Simplify:

a) 5(3*x* – 7) – 6*x*
b) 3(1+y)-2xy+2(x+y)

Linear Equations in One Variable and Proportions

A **linear equation in one variable *x*** is an equation that can be written in the form $ax + b = 0$, where ***a*** and ***b*** are real numbers, and $a \neq 0$.

Solving an equation in *x* involves determining all values of *x* that result in a true statement when substituted into the equation. Such values are **solutions** or **roots**.

Equivalent equations have the same solution set.

$a = b$ and $a + c = b + c$ are equivalent equations.

$a = b$ and $ac = bc$ are equivalent equations.

Example: Solve and check: 2(*x* – 4) – 5*x* = –5.

Solve: 2*x* + 6 = 2(*x* + 4)

Solve: 4*x* + 6 = 6(*x* + 1) – 2*x* The solution set is the set of all real numbers, expressed as {x|x is a real number}.

Equations and Inequalities

A linear inequality :

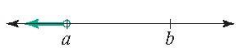
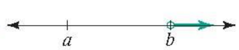
$ax + b \leq c$

where the inequality symbol can be <, >, ≤, or ≥.

Solving an inequality is the process of finding the set of numbers that make an inequality a true statement.

A solution set is the set of all numbers that satisfy the inequality.

Graphing Subsets of Real Numbers on a Number Line

Let *a* and *b* be real numbers such that $a < b$.		
Set-Builder Notation		**Graph**
$\{x \mid x < a\}$	x is a real number less than a.	
$\{x \mid x \leq a\}$	x is a real number less than or equal to a.	
$\{x \mid x > b\}$	x is a real number greater than b.	
$\{x \mid x \geq b\}$	x is a real number greater than or equal to b.	
$\{x \mid a < x < b\}$	x is a real number greater than a and less than b.	
$\{x \mid a \leq x \leq b\}$	x is a real number greater than or equal to a and less than or equal to b.	
$\{x \mid a \leq x < b\}$	x is a real number greater than or equal to a and less than b.	
$\{x \mid a < x \leq b\}$	x is a real number greater than a and less than equal to b.	

The procedure for solving linear inequalities is the same as the procedure for solving linear equations, with one important exception: When multiplying or dividing both sides of the inequality by a negative number, reverse the direction of the inequality symbol, changing the sense of the inequality.

Solve and graph the solution set: $6x - 12 > 8x + 2$

Solve the three part inequality:

$$
\begin{aligned}
-3 &< 2x+1 \le 3\\
-3-1 &< 2x+1-1 \le 3-1\\
-4 &< 2x \le 2\\
\frac{-4}{2} &< \frac{2x}{2} < \frac{2}{2}\\
-2 &< x \le 1
\end{aligned}
$$

The solution set is $\{\, x \mid -2 < x \le 1\}$.

Graphing and Functions

A relationship between two quantities can be expressed as an ***equation in two variables***, such as

$y = 4 - x^2$.

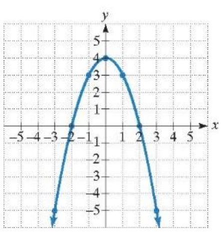
A **solution of an equation in two variables**, x and y, is an ordered pair of real numbers with the following property:

When the x-coordinate is substituted for x and the y coordinate is substituted for y in the equation, we obtain a true statement.

The ***graph of an equation in two variables*** is the set of all points whose coordinates satisfy the equation.

Graph $y = 4 - x^2$. Select integers for x, starting with −3 and ending with 3.

Now plot the seven points and join them with a smooth curve.



Functions

If an equation in two variables (x and y) yields precisely one value of y for each value of x, we say that y is a ***function*** of x.

The notation $y = f(x)$ indicates that the variable y is a function of x. The notation $f(x)$ is read "*f* of *x*."

Example: Graph the functions $f(x) = 2x$ and $g(x) = 2x + 4$ in the same rectangular coordinate system. Select integers for x from −2 to 2, inclusive.

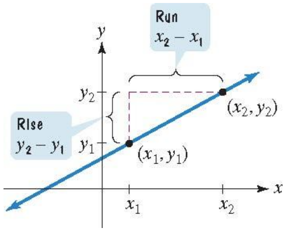
Graphs, Functions, and Linear Systems

All equations of the form $Ax + By = C$ are straight lines when graphed, as long as A and B are not both zero, and are called *linear equations in two variables*.

Example: Graph: $3x + 2y = 6$.

The ***slope*** of the line through the distinct points (x_1,y_1) and (x_2,y_2) is

$$\frac{\text{Change in } y}{\text{Change in } x} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{y_2 - y_1}{x_2 - x_1}$$



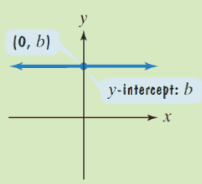
Graphing $y = mx + b$ using the slope and y-intercept.

Example: Graph the linear function by using the slope and y-intercept.

$$y = \frac{2}{3}x + 2$$

HORIZONTAL AND VERTICAL LINES

The graph of $y = b$ or $f(x) = b$ is a horizontal line. The y-intercept is b.



The graph of $x = a$ is a vertical line. The x-intercept is a.