Assignment 03 answers

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Suggested solutions for selected exercises chapter 3 of Pure Mathematics 1 by Hugh Neil and Douglas Qualing.

1.

The doming of the function is the set of numbers for which the function is defined.

a) $\dfrac{1}{1+\sqrt{x}}$ Since the square root of a negative number is not defined in the set of real numbers, the domain of this function is all x greater or equal to zero: $x \geq 0$.

b) $\dfrac{1}{(x-1)(x-2)}$; the divisibility by zero has no meaning; or, we say, it is undefined. The answer can be stated as follow: Domain of the function is all x, such as $x \neq 1$ and $x \neq 2$.

2.

The Domain of these functions are the set of **all positive real numbers**. That is, negative numbers are not considered in the input (as admissible values for x). The range of the given function are:

a) for $f(x) = -5x$ Since the input we consider are positive values of x, the output will always be negative. Answer: $y < 0$.

b) for $f(x) = 3x - 1$; since the input we consider are positive values of x, being zero the smallest, the output will be $y > -1$.

c) for $f(x) = (x-1)^2 + 2$, $y \geq 2$.

3.

Note: when we take the reciprocal of both sides of an inequality, the sign of the inequality is reversed provided $a, b, c, d > 0$. And this is why:

$\dfrac{a}{b} > \dfrac{c}{d}$

Multiply both sides of the inequality by bd:

$ad > bc$

Divide both sides of the inequality by ac:

$\dfrac{d}{c} > \dfrac{b}{a}$ This result is equivalent to:

$\dfrac{b}{a} < \dfrac{d}{c}$ which is the reciprocal of the original inequality and the inequality sign has been inverted.

a) $x^{-4} \geq 100$ it is the same as: $\dfrac{1}{x^4} \geq 10^2$

Taking the reciprocal of the inequality:

$x^4 \leq \frac{1}{10^2}$

We need to know that, the nth (even) root of a number is the modulus of the number: $\sqrt[n]{x} = |x|$, again, provided n is even. So, in our case we get:

$|x| \leq \frac{\sqrt{10}}{10}$ which leads to:

$-\frac{\sqrt{10}}{10} \leq x \leq \frac{\sqrt{10}}{10}$.

b) $8x^{-4} < 0.00005$.

$8x^{-4} < 5 \cdot 10^{-5}$

$\frac{8}{x^4} < \frac{5}{10^5}$

$\frac{x^4}{8} > \frac{10^5}{5}$

$x^4 > \frac{8 \cdot 10^5}{5}$

$x^4 > 2^4 \cdot 10^4$

$|x| > 20$

Therefore $x > 20$ or $-x > 20$ which is equivalent to: $x > 20$ or $x < -20$.

4.

In order to sketch the graphs of the functions, draw a xy coordinate system using k as a unit. This is, in the positive side of both axes (x and y) we have k, $2k$, $3k$, etc; on the negatives, $-k$, $-2k$, $-3k$ etc. Also, let's pay attention to the highest power of x and the sign (positive or negative) of the highest coefficient of x.
Then, in a)the parabola opens upwards, the zeros are $-4k$ and $-2k$.
b) The power of x is 3; so the curve takes the shape of the parent function $y = x^3$ with zeros at 0, k and $5k$.

5.

We are given the equation of three curves (parabolas). Let's label them as y_1, y_2 and y_3. All three curves have one point in common. We set pair of equations in order to determine the point in common between two of them, as follows: y_1 and y_2; then y_1 and y_3 and finally, y_2 and y_3, which are all possible combinations.
Equating y_1 and y_2 we get:

$$2x^2 + 5x = x^2 + 4x + 12 \;\; or \;\; x^2 + x - 12 = 0 \;\; which \; factorizes \; as \; (x+4)(x-3) = 0$$

Therefore the two curves have two x coordinate in common: $x = -4$ and $x = 3$.
let's pair y_1 and y_3:

$$2x^2 + 5x = 3x^2 + 4x - 6$$
$$x^2 - x - 6 = 0$$

Whose factors are: $(x-3)(x+2)=0$ given two x-values in common between these two curves: $x=3$ and $x=-2$. Indeed we have found that $x=3$ is a common x-coordinate; still, let's verify that y_2 and y_3 have the same point in common:

$$x^2+4x+12=3x^2+4x-6$$
$$2x^2-18=0$$

Or $x=\pm3$. Again, we have found that $x=3$ is a common value for all three curves. The y-coordinate can be calculated by substituting in either of the original equations; for instance $f(3)=2(3)^2+5(3)=33$. The common point is $(3,33)$.

6.

Since both curves meet at $(-2,12)$ by evaluating $f(-2)=12$ in the equations we can determine the values of c and k:

$$f(-2)=12=(-2)^2-3(-2)+c \quad or \quad c=2$$
$$f(-2)=12=k-(-2)-(-2)^2 \quad or \quad k=14$$

Therefore our functions are:

$$y=x^2-3x+2 \quad and \quad y=14-x-x^2$$

In order to determine the other point at which the two curves meet, we set them equal to one another:

$$14-x-x^2=x^2-3x+2$$
$$2x^2-2x-12=0$$
$$(x-3)(x+2)=0 \quad therefore \quad x_1=-2 \; and \quad x_2=3.$$

$x_1=-2$ was already known to us; so the other point has y-coordinate $=2$ since $f(3)=2$, the point is $(3,2)$.

7.

The straight line $y=x-1$ meets the curve $y=x^2-5x-8$ at the points A and B. The curve $y=p+qx-2x^2$ also passes through A and B. We are asked to find the values of p and q.
Setting the two first equations equal to each other, we are able to find A and B:

$$x-1=x^2-5x-8 \quad which\ reduces\ to\ x^2-6x-7=0 \quad or\ (x-7)(x+1)=0$$

The x-values at which the curves meet are: $x_1=7$ and $x_2=-1$. Since $f(7)=6$ and $f(-1)=-2$ points A and B are: $(7,6)$ and $(-1,-2)$.
Now, by evaluating those points in $y=p+qx-2x^2$ we get:

$$f(7)=6=p+7q-2(7)^2 \quad or \quad p+7q=104$$
$$f(-1)=-2=p+q(-1)-2(-1)^2 \quad or \quad p-q=0$$

By solving this system of two unknowns, we get: $8p=104$ or $p=13$, therefore $q=13$.

8.

In order to the points at which the line $y10x-9$ meets the curve $y=x^2$, again we set the equation equal to each other:

$$10x-9=x^2$$
$$x^2-10+9=0$$
$$(x-9)(x-1)=0$$

Therefore $x_1=9$ and $x_2=1$. $f(9)=81$ and $f(1)=9$. The points of intersection are: $(9,81)$ and $(1,9)$.