

## Selected questions

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Questions taken from Pure Mathematics 1, by Hugh Neil and Douglas Quailing, Cambridge University Press, 2002.

### Miscellaneous exercises 1, page 15, No. 1.

1. Show that the triangle formed by the points  $(-2,5)$ ,  $(1, 3)$  and  $(5, 9)$  is right-angled.

### Miscellaneous exercises 4, page 62, No. 4

4. For what values of  $k$  does the equation  $2x^2 - kx + 8 = 0$  have a repeated root?

### Exercises 6D, page 87, No. 12

12. A curve has equation  $y = x(x - a)(x + a)$ , where  $a$  is a constant. Find the equations of the tangents to the graph at the points where it crosses the  $x$ -axis.

### Exercise 6E, page 93, No. 2

2. Find the derivative of the function  $f(x) = x^8$  at  $x = p$ . (Use the difference of two squares formula on  $q^8 - p^8$  as often as you can.)

### Miscellaneous exercises 6, page 94, No.11

11. Show that the curves  $y = x^3$  and  $y = (x + 1)(x^2 + 4)$  have exactly one point in common, and use differentiation to find the gradient of each curve at this point.

### Exercises 7C, page 110, No.16

16. A circular cylinder is to fit inside a sphere of radius 10 cm . Calculate the maximum possible volume of the cylinder. (It is probably best to take as your independent variable the height, or half the height, of the cylinder.)

### Miscellaneous exercises 8, page 126 No. 8 and 9

8. An arithmetic progression has first term  $a$  and common difference  $-1$ . The sum of the first  $n$  terms is equal to the sum of the first  $3n$  terms. Express  $a$  in terms of  $n$ .

9. Find the sum of the arithmetic progression 1, 4, 7, 10, 13, 16,,1000. Every third term of the above progression is removed, i.e. 7, 16, etc. Find the sum of the remaining terms.

### Miscellaneous exercises 9, page 136 No. 17 and No. 28 page 137.

17. Find and simplify the term independent of  $x$  in the expansion of  $\left(\frac{1}{2x} + x^3\right)^8$ .

28. Prove that  $\binom{n}{r-1} + 2\binom{n}{r} + \binom{n}{r+1} = \binom{n+2}{r+1}$

**Miscellaneous exercises 11, page 173 No. 19**

19. Given the function  $f(x) = \frac{2x-4}{x}$   $x \in \mathbf{R}$  and  $x \neq 0$ ., find:

- (a)  $f^2(x)$  (b)  $f^{-1}(x)$  (c)  $f^4(x)$  (d)  $f^{10}(x)$  (e)  $f^{12}(x)$  (f)  $f^{82}$

(The notation  $f^2(x)$  represent the composite function  $ff$ ;  $f^3(x)$ , represents  $fff$ , and so on.)

**Miscellaneous exercises 9, page 137**

32. Find an expression, in terms of n, for the coefficient of  $x$  in the expansion  $(1+4x)+(1+4x)^2+(1+4x)^3+(1+4x)^4+\dots$

**Exercise 13D, page 207**

16. The roof of a house has a rectangular base of side 4 metres by 8 metres. The ridge line of the roof is 6 metres long, and centred 1 metre above the base of the roof. Calculate the acute angle between two opposite slanting edges of the roof.