Exercise 6D

1 Differentiate the following functions.

(a) x^3+2x^2 (b) $1-2x^3+3x^2$ (c) $x^3-6x^2+11x-6$

(d) $2x^3-3x^2+x$ (e) $2x^2(1-3x^2)$ (f) $(1-x)(1+x+x^2)$

2 Find $f'(-2)$ for each of the following functions $f(x)$.

(a) $2x-x^3$ (b) $2x-x^2$ (c) $1-2x-3x^2+4x^3$

(d) $2-x$ (e) $x^2(1+x)$ (f) $(1+x)(1-x+x^2)$

3 For each of the following functions $f(x)$ find the value(s) of x such that $f'(x)$ is equal to the given number.

(a) x^3 12 (b) x^3-x^2 8 (c) $3x-3x^2+x^3$ 108

(d) x^3-3x^2+2x −1 (e) $x(1+x)^2$ 0 (f) $x(1-x)(1+x)$ 2

4 Differentiate the following functions.

(a) $2\sqrt{x}$ (b) $\left(1+\sqrt{x}\right)^2$ (c) $y=x-\frac{1}{2}\sqrt{x}$ (d) $x\left(1-\frac{1}{\sqrt{x}}\right)^2$

(e) $x-\frac{1}{x}$ (f) $\frac{x^3+x^2+1}{x}$ (g) $\frac{(x+1)(x+2)}{x}$ (h) $\left(\frac{\sqrt{x}+x}{\sqrt{x}}\right)^2$

5 Find the equation of the tangent to the curve $y=x^3+x$ at the point for which $x=-1$.

6 One of the tangents to the curve with equation $y=4x-x^3$ is the line with equation $y=x-2$. Find the equation of the other tangent parallel to $y=x-2$.

7 Find the equation of the tangent at the point $(4,2)$ to the curve with equation $y=\sqrt{x}$.

8 Find the equation of the tangent at the point $\left(2,\frac{1}{2}\right)$ to the curve with equation $y=\frac{1}{x}$.

9 Find the equation of the normal at the point $(1,2)$ to the graph $y=x+\frac{1}{x}$.

10 The graphs of $y=x^2-2x$ and $y=x^3-3x^2-2x$ both pass through the origin. Show that they share the same tangent at the origin.

11 Find the equation of the tangent to the curve with equation $y=x^3-3x^2-2x-6$ at the point where it crosses the y-axis.

12 A curve has equation $y=x(x-a)(x+a)$, where a is a constant. Find the equations of the tangents to the graph at the points where it crosses the x-axis.

13 Find the coordinates of the point of intersection of the tangents to the graph of $y=x^2$ at the points at which it meets the line with equation $y=x+2$.