

Miscellaneous exercise 4

- 1 Solve the simultaneous equations $x + y = 2$ and $x^2 + 2y^2 = 11$.
- 2 The quadratic polynomial $x^2 - 10x + 17$ is denoted by $f(x)$. Express $f(x)$ in the form $(x - a)^2 + b$ stating the values of a and b .
Hence find the least possible value that $f(x)$ can take and the corresponding value of x .
- 3 Solve the simultaneous equations $2x + y = 3$ and $2x^2 - xy = 10$.
- 4 For what values of k does the equation $2x^2 - kx + 8 = 0$ have a repeated root?
- 5 By expressing the function $f(x) = (2x + 3)(x - 4)$ in completed square form, find the range of the function $f(x)$.
- 6 (a) Solve the equation $x^2 - (6\sqrt{3})x + 24 = 0$, giving your answer in terms of surds, simplified as far as possible.
(b) Find all four solutions of the equation $x^4 - (6\sqrt{3})x^2 + 24 = 0$ giving your answers correct to 2 decimal places.
- 7 Show that the line $y = 3x - 3$ and the curve $y = (3x + 1)(x + 2)$ do not meet.
- 8 Express $9x^2 - 36x + 52$ in the form $(Ax - B)^2 + C$, where A , B and C are integers.
Hence, or otherwise, find the set of values taken by $9x^2 - 36x + 52$ for real x .
- 9 Find the points of intersection of the curves $y = 6x^2 + 4x - 3$ and $y = x^2 - 3x - 1$, giving the coordinates correct to 2 decimal places.
- 10 (a) Express $9x^2 + 12x + 7$ in the form $(ax + b)^2 + c$ where a , b , c are constants whose values are to be found.
(b) Find the set of values taken by $\frac{1}{9x^2 + 12x + 7}$ for real values of x .
- 11 Find, correct to 3 significant figures, all the roots of the equation $8x^4 - 8x^2 + 1 = \frac{1}{2}\sqrt{3}$.
- 12 Find constants a , b and c such that, for all values of x ,
$$3x^2 - 5x + 1 = a(x + b)^2 + c.$$

Hence find the coordinates of the minimum point on the graph of $y = 3x^2 - 5x + 1$.
(Note: the minimum point or maximum point is the vertex.)
- 13 Find the points of intersection of the curve $xy = 6$ and the line $y = 9 - 3x$.