Miscellaneous exercise 4

1 Solve the simultaneous equations \( x + y = 2 \) and \( x^2 + 2y^2 = 11 \).

2 The quadratic polynomial \( x^2 - 10x + 17 \) is denoted by \( f(x) \). Express \( f(x) \) in the form \( (x - a)^2 + b \) stating the values of \( a \) and \( b \).
Hence find the least possible value that \( f(x) \) can take and the corresponding value of \( x \).

3 Solve the simultaneous equations \( 2x + y = 3 \) and \( 2x^2 - xy = 10 \).

4 For what values of \( k \) does the equation \( 2x^2 - kx + 8 = 0 \) have a repeated root?

5 By expressing the function \( f(x) = (2x + 3)(x - 4) \) in completed square form, find the range of the function \( f(x) \).

6 (a) Solve the equation \( x^2 - (6\sqrt{3})x + 24 = 0 \) giving your answer in terms of surds, simplified as far as possible.
(b) Find all four solutions of the equation \( x^4 - (6\sqrt{3})x^2 + 24 = 0 \) giving your answers correct to 2 decimal places.

7 Show that the line \( y = 3x - 3 \) and the curve \( y = (3x + 1)(x + 2) \) do not meet.

8 Express \( 9x^2 - 36x + 52 \) in the form \( (Ax - B)^2 + C \), where \( A \), \( B \) and \( C \) are integers.
Hence, or otherwise, find the set of values taken by \( 9x^2 - 36x + 52 \) for real \( x \).

9 Find the points of intersection of the curves \( y = 6x^2 + 4x - 3 \) and \( y = x^2 - 3x - 1 \), giving the coordinates correct to 2 decimal places.

10 (a) Express \( 9x^2 + 12x + 7 \) in the form \( (ax + b)^2 + c \) where \( a \), \( b \), \( c \) are constants whose values are to be found.
(b) Find the set of values taken by \( \frac{1}{9x^2 + 12x + 7} \) for real values of \( x \).

11 Find, correct to 3 significant figures, all the roots of the equation \( 8x^4 - 8x^2 + 1 = \frac{1}{2} \sqrt{3} \).

12 Find constants \( a \), \( b \) and \( c \) such that, for all values of \( x \),
\[ 3x^2 - 5x + 1 = a(x + b)^2 + c. \]
Hence find the coordinates of the minimum point on the graph of \( y = 3x^2 - 5x + 1 \).
(Note: the minimum point or maximum point is the vertex.)

13 Find the points of intersection of the curve \( xy = 6 \) and the line \( y = 9 - 3x \).