Practice 9, Questions

Miscellaneous exercises 6, questions 7 to 15, page 94, of Pure Mathematics 1 by Hugh Neil and Douglas Qualing.

7. Find the equation of the normal to $f(x) = (2x - 1)(3x + 5)$ at the point $(1, 8)$. Give your answer in he form $ax + by + c = 0$, where $a, bandc$ are integers.

8. The curve $f(x) = x^2 - 3x - 4$ crosses the x-axis at P and Q. The tangents to the curves at P and Q meet at R. The normals to the curve at P and Q meet at S. Find the distance RS.

9. The equation of a curve is $y = 2x^2 - 5x + 14$. The normal to the curve at the point $(1, 11)$ meets the curve again at the point P. Find the coordinate of P.

10. At a particular point of the curve $f(x) = x^2 + k$, the equation of the tangent is $y = 6x - 7$. Find the value of the constant k.

11. Show that the curves $y = x^3$ and $y = (x + 1)(x^2 + 4)$ have exactly one point in common, and use differentiation to find the gradient of each curve at this point.

12. At a particular point of the curve $y = 5x^2 - 12x + 1$ the equation of the normal is $x + 18y + c = 0$. Find the constant c.

13. The graphs of $y = x^m$ and $y = x^n$ intersect at the point $P(1, 1)$. Find the connection between m and n if the tangent at P to each curve is the normal to the other curve.

14. The tangent at $x = \frac{1}{4}$ to $y = \dfrac{1}{\sqrt{x}}$ meet at P. Find the coordinates of P.

14. The tangents at $x = \dfrac{1}{4}$ to $y = \sqrt{x}$ and $y = \dfrac{1}{\sqrt{x}}$ meet at P. Find the coordinates of P.

15. The normals at $x = 2$ to $y = \dfrac{1}{x^2}$ and $y = \dfrac{1}{x^3}$ meet at Q. Find the coordinates of Q.