Practice 9, Questions

Miscellaneous exercises 6, questions 7 to 15, page 94, of Pure Mathematics 1 by Hugh Neil and Douglas Qualing.

7. Find the equation of the normal to \( f(x) = (2x - 1)(3x + 5) \) at the point (1,8). Give your answer in the form \( ax + by + c = 0 \), where \( a, b, c \) are integers.

8. The curve \( f(x) = x^2 - 3x - 4 \) crosses the x-axis at P and Q. The tangents to the curves at P and Q meet at R. The normals to the curve at P and Q meet at S. Find the distance RS.

9. The equation of a curve is \( y = 2x^2 - 5x + 14 \). The normal to the curve at the point (1,11) meets the curve again at the point P. Find the coordinate of P.

10. At a particular point of the curve \( f(x) = x^2 + k \), the equation of the tangent is \( y = 6x - 7 \). Find the value of the constant \( k \).

11. Show that the curves \( y = x^3 \) and \( y = (x + 1)(x^2 + 4) \) have exactly one point in common, and use differentiation to find the gradient of each curve at this point.

12. At a particular point of the curve \( y = 5x^2 - 12x + 1 \) the equation of the normal is \( x + 18y + c = 0 \). Find the constant \( c \).

13. The graphs of \( y = x^m \) and \( y = x^n \) intersect at the point \( P(1,1) \). Find the connection between \( m \) and \( n \) if the tangent at \( P \) to each curve is the normal to the other curve.

14. The tangent at \( x = \frac{1}{4} \) to \( y = \frac{1}{\sqrt{x}} \) meet at \( P \). Find the coordinates of \( P \).

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15. The normals at \( x = 2 \) to \( y = \frac{1}{x^2} \) and \( y = \frac{1}{x^3} \) meet at \( Q \). Find the coordinates of \( Q \).