Practice 14

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Suggested solutions for Miscellaneous exercises 10, problems 5-20, pages 153 to 155 from Pure Mathematics 1, by Hugh Neil and Douglas Quailing, Cambridge University Press, 2002.

5.

Redefine $2x$ as ϕ; the x interval $0 \leq x \leq 180$ becomes $0 \leq 2x \leq 360$ that is $0 \leq \phi \leq 360$. Then, $3cos\phi = 2$ implies $cos\phi = \dfrac{2}{3}$; therefore, $cos^{-1}(\frac{2}{3}) = \phi = 48.20$ (using the cos^{-1} key in scientific a calculator). Now, since $cos(-\theta) = cos(\theta)$, then -48.20 is another root. Applying the periodic property of $cos(\theta \pm 360) = cos(\theta)$ we have $-48.20 + 360 = 311.80$. (Adding another 360° will take the angle outside the given interval ($0 \leq 2x \leq 360$). So, we have $\phi = 48.20°$ and $\phi = 311.80°$; the answer in terms of $x°$ is given by $2x° = \phi$; therefore, the two solutions in the interval are: $x° = \frac{1}{2}\phi$ or $x° = 24.1$ and $x° = 155.9$.

6.

(a) Period of the tangent and cotangent functions is 180°; sine —$sin(bx°)$, and cosine, $cosbx°$, have period $T = \dfrac{360}{b}$; hence, when $b = 2$, $sin2x°$ and $cos2x°$ both have period equal to 180°

(b) $sin3x° = 0.5$; interval, $0 < x < 180$; redefine $3x° = \phi$ that is, ϕ interval is three times $x°$ interval; that is, $0 < \phi < 540$. We know that the angle whose sine is 0.5 is 30°; or, using the calculator, $sin^{-1}(0.5) = 30°$; another solution is based on the property $sin(180 - \theta) = sin(\theta)$ which implies that $180 - 30 = 150°$ is another solution. Considering that ϕ interval upper limit is 540°, and applying the periodic property, we have two additional solutions: $30 + 360 = 390$ and $150 + 360 = 510$; then, solving for $x° = \frac{1}{3}\phi$ we have: $x_1 = 10°$; $x_2 = 50°$; $x_3 = 130°$ and $x_4 = 170°$.

7.

Given the interval: $0 \leq \theta \leq 360$ and the equation $2cos(\theta + 30)° = 1$ values of θ in the interval:
make $\phi = \theta + 30$; our interval becomes $30 \leq \phi \leq 390$. The equation $2cos(\phi) = 1$ simplifies to $cos(\phi) = 0.5$; the angle whose cosine is 0.5 is 60°: $cos^{-1}(0.5) = 60°$. Another solution for the cosine function is given by $cos(-\theta) = cos(\theta)$; however, in our case, $-60°$ in not in the interval under consideration. Let's apply the periodic property, therefore, $-60 + 360 = 300°$. $(60 + 360 = 420°$ is outside the interval). Then, solving for $\theta = \phi - 30$ we have two solutions for the equation: $x_1 = 30°$ and $x_2 = 270°$.

8.

(a) We know that the cofunctions of complementary angles are equal to each other:
$sin(90 - \theta) = cos(\theta)$ and $cos(90 - \theta) = sin(\theta)$; that is $cos(90 - 2x) = sin(2x)$ which implies that $sin2x + cos(90 - 2x) = sin2x + sin2x = 2sin(2x)$.
(b) Hence, $2sin(2x) = -1$. Now, we define $\phi = 2x$, and the interval $0 \leq x \leq 360$ becomes $0 \leq \phi \leq 720$. Then, $sin(\phi) = -\frac{1}{2}$ and $sin^{-1}(-0.5) = -30°$; which is not in the interval; applying the periodic property of the sine function we get:$-30 + 360 = 330°$ and, considering that $sin(180 - \theta) = sin(\theta)$another solution is $180 - (-30) = 210°$; in addition to these two solutions (300° and 210°) in the interval $0° to 360°$ we have to consider adding 360° —periodic property— since our modified interval spreads from $0° to 720°$; that is: $330 + 360 = 690°$ and $210 + 360 = 570°$. Now, solving for $x°$ we have: $x° = \frac{1}{2}\phi$ which yields $x_1 = 105°$, $x_2 = 165°$, $x_3 = 285°$ and $x_4 = 345°$.

9.

We have o find the least positive value —that is, zero is not included— of the angle A for which:

(a) $sinA° = 0.2$ and $cosA° < 0$; sine is positive; cosine is negative: II quadrant. Since $sinA° = 0.2$; $sin^{-1} = 11.5°$ which is located in the I quadrant; in the second quadrant we have: $sin(180 - \theta) = sin(\theta)$; that is, $180 - 11.5 = 168.5°$.
(b) $tanA° < 0$, $sinA° < 0$: IV quadrant.
$tanA° = -0.5$; therefore, $tan^{-1}(-0.5) = -26.6$; this result is not *the least positive* value, it is indeed, negative; but $-26.6°$ is coterminal with $-26.2 + 360 = 333.4$. (c)$cosA° = sinA°$ both negative. III quadrant. Now, in the I quadrant we know that $cosA° = sinA° = \frac{\sqrt{2}}{2}$ for $A° = 45°$; in this case both, sine and cosine are negative, therefore $cosA° = sinA° = -\frac{\sqrt{2}}{2}$ and since $sin^{-1}(-\frac{\sqrt{2}}{2}) = -45°$ applying $sin(180 - \theta) = sin(\theta)$ we get $180 - (-45) = 225°$.

(d) $sinA° = -0.2275$ $A° > 360$ by calculator $sin^{-1}(-0.2275) = -13.15°$ The smallest angle whose sine is negative lies in the III quadrant, that is $sin(180 - \theta) = sin(\theta)$, then $180 - (-13.15) = 193.15$; but it is given that $A° > 360$ thus, applying the periodic property, the answer is: $193.15 + 360 = 553.15°$.

10.

(a) Proving that: $\dfrac{1}{sin\theta} - sin\theta \equiv \dfrac{cos\theta}{tan\theta}$. Combining the two terms in the left hand side:
$\dfrac{1 - sin^2\theta}{sin\theta} \equiv \dfrac{cos\theta}{tan\theta}$; from the Pythagorean identity we have: $cos^2\theta = 1 - sin^2\theta$; then,
$\dfrac{cos^2\theta}{sin\theta} \equiv \dfrac{cos\theta}{tan\theta}$; rewriting $cos^2\theta$ as $cos\theta \cdot cos\theta$ realizing that since $tan\theta = \dfrac{sin\theta}{cos\theta}$ then $\dfrac{cos\theta}{sin\theta} = \dfrac{1}{tan\theta}$ we get:
$\dfrac{cos\theta \cdot cos\theta}{sin\theta} \equiv \dfrac{cos\theta}{tan\theta}$ that is:
$cos\theta \cdot \dfrac{cos\theta}{sin\theta} \equiv \dfrac{cos\theta}{tan\theta}$ which leads to: $\dfrac{cos\theta}{tan\theta} \equiv \dfrac{cos\theta}{tan\theta}$.

(b) $\dfrac{1 - sin\theta}{cos\theta} \equiv \dfrac{cos\theta}{1 + sin\theta}$

Let's multiply the left hand side by $\dfrac{(1 + sin\theta)}{(1 + sin\theta)}$ which produces a difference of two squares $1 - sin^2\theta$ in the numerator:
$\dfrac{1 - sin\theta}{cos\theta} \cdot \dfrac{(1 + sin\theta)}{(1 + sin\theta)} \equiv \dfrac{cos\theta}{1 + sin\theta}$
$\dfrac{1 - sin^2\theta}{cos\theta \cdot (1 + sin\theta)} \equiv \dfrac{cos\theta}{1 + sin\theta}$ considering that, by the Pythagorean identity, $cos^2\theta = 1 - sin^2\theta$, we have:
$\dfrac{cos^2\theta}{cos\theta \cdot (1 + sin\theta)} \equiv \dfrac{cos\theta}{1 + sin\theta}$ which simplifies to:
$\dfrac{cos\theta}{(1 + sin\theta)} \equiv \dfrac{cos\theta}{1 + sin\theta}$

(c) $\dfrac{1}{tan\theta} + tan\theta \equiv \dfrac{1}{sin\theta\ cos\theta}$ combining the two terms on the left hand side:
$\dfrac{1 + tan^2\theta}{tan\theta} \equiv \dfrac{1}{sin\theta\ cos\theta}$ we know that $1 + tan^2\theta = sec^2\theta$; then,
$\dfrac{sec^2\theta}{tan\theta} \equiv \dfrac{1}{sin\theta\ cos\theta}$ substituting $\dfrac{1}{cos^2\theta} = sec^2\theta$ and $\dfrac{sin\theta}{cos\theta} = tan\theta$, the left hand side becomes:
$\dfrac{1}{cos^2\theta \cdot \dfrac{sin\theta}{cos\theta}} \equiv \dfrac{1}{sin\theta\ cos\theta}$ which simplifies to:
$\dfrac{1}{sin\theta\ cos\theta} \equiv \dfrac{1}{sin\theta\ cos\theta}$

(d) $\dfrac{1 - 2sin^2\theta}{cos\theta + sin\theta} \equiv cos\theta - sin\theta$ the left hand side can be rewritten as:

$\dfrac{1 - sin^2\theta - sin^2\theta}{cos\theta + sin\theta} \equiv cos\theta - sin\theta$; therefore, considering that $1 - sin^2\theta = cos^2\theta$:

$\dfrac{cos^2\theta - sin^2\theta}{cos\theta + sin\theta} \equiv cos\theta - sin\theta$; the numerator on the left hand side, a difference of two squares, becomes:

$\dfrac{(cos\theta - sin\theta)(cos\theta + sin\theta)}{cos\theta + sin\theta} \equiv cos\theta - sin\theta$; which simplifies to:

$cos\theta - sin\theta \equiv cos\theta - sin\theta$

11.

The maximum and minimum values of y and the least positive values of x at which these occur:
Note: the cosine and the sine functions ($y = cos\theta$, $y = sin\theta$) reach a minimum value $y = -1$ and a maximum value $y = 1$; in summary, the range of both functions is $-1 \leq y \leq 1$.

(a) $y = 1 + cos2x$ reaches a maximum $y = 2$ whenever $cos2x = 1$; therefore, x° value is given by: $cos^{-1}(1) = 0$ but we cannot consider zero as a valid solution since we are looking for *the least positive values of* x; then $0^\circ + 360^\circ = 360^\circ$, thus $2x = 360^\circ$ and $x = 180^\circ$.
$y = 1 + cos2x$ reaches a minimum value of $y = 0$ whenever $cos2x = -1$; and it occurs at: $cos^{-1}(-1) = 180^\circ$ therefore, $2x = 180^\circ$ or $x = 90^\circ$.

(b) $y = -4sin(x + 30)$; maximum, $y = 9$ occurs at $sin(x + 30) = -1$ and since $sin^{-1}(-1) = 270^\circ$ the x values is given by: $x + 30 = 270^\circ$ or $x = 240^\circ$.
The minimum value of $y = 1$ occurs at $sin(x+30) = 1$; that is, $sin^{-1}(1) = 90^\circ$ which implies that $x+30 = 90^\circ$ or $x = 60^\circ$.

(c) $y = 29 - 20sin(3x - 45)^\circ$ reaches a maximum value of $y = 49$ when $sin(3x - 45) = -1$; the x° value is given by: $sin^{-1}(-1) = 270^\circ$ that is $3x - 45 = 270^\circ$ or $x = \dfrac{270 + 45}{3} = 105^\circ$.
Minimum value of $y = 9$ occurs when $sin(3x - 45) = 1$; in this case the x° value is given by: $sin^{-1}(1) = 90^\circ$; which implies that $x = 45^\circ$.

(d) $y = 8 - 3cos^2x$ since $cos^2x > 0$ both, $(-1)^2 = (1)^2 = 1$; the maximum value of $y = 8$ is reached when $cosx = 0$. And, $cos^{-1}(0) = 90^\circ$; hence, $x = 90^\circ$.
The minimum value of $y = 5$ occurs when $cos^2x = 1$; which implies that $cosx = |1|$ or $cosx = \pm 1$; since $cos^{-1}(1) = 360^\circ$ (least positive value) and $cos^{-1}(-1) = 180^\circ$ therefore, the least positive value of x is 180°.

(e) $y = \dfrac{12}{3 + cosx}$ this time, the maximum value of $y = 6$ is reached when the denominator takes its smallest vale at $cosx = -1$; that is at $cos^{-1}(-1) = x = 180^\circ$. The minimum value of $y = 3$ occurs when $cosx = 1$ therefore $cos^{-1}(1) = x = 0^\circ$. Considering the periodic property of the cosine function the *the least positive values of* x is $0^\circ + 360^\circ = 360^\circ$.

(f) $y = \dfrac{60}{1 + sin^2(2x - 15)}$ The function reaches its maximum value, $y = 60$ when $sin^2(2x-15) = 0$; that is, $sin^{-1}(0) = x = 0^\circ$. Solving for x we have: $2x - 15 = 0$; hence $x = \dfrac{15}{2} = 7.5^\circ$.
The minimum value $y = 30$ occurs when $sin(2x - 15) = \pm 1$; therefore, there are two possibilities: $sin^{-1}(-1) = x = -90^\circ$. and $sin^{-1}(1) = x = 90^\circ$. We takes the least positive vale, 90°; then, $2x - 15 = 90$; hence, $x = \dfrac{15 + 90}{2} = 52.5^\circ$.

12.

(a) $sin\theta = tan\theta$ The equality, $sin\theta = \dfrac{sin\theta}{cos\theta}$ is true when $cos\theta = 1$ or $sin\theta = 1$; and it occurs when $cos^{-1}(1) = x^\circ = 0^\circ$ or $sin^{-1}(0) = x^\circ = 0^\circ$ and $x^\circ = 360^\circ$.

(b) $2 - 2cos^2\theta = sin\theta$ considering the identity $sin^2\theta + cos^2\theta = 1$; or $cos^2\theta = 1 - sin^2\theta$, and substituting, we obtain:

$2 - 2(1 - sin^2\theta) = sin\theta$ equivalent to:

$2-2-2sin^2\theta = sin\theta$ which reduces to: $-2sin^2\theta = sin\theta$ equating it to zero in order to factorize the equation:

$$0 = sin\theta - 2sin^2\theta$$
$$0 = sin\theta(1-2sin\theta)$$

Therefore, either $sin\theta = 0$ or $1-2sin\theta = 0$; in the first instance we have as solution $sin^{-1}(0) = 0°$ also $sin\theta = 0°$ in the given interval at $x = 180°$ and $x = 360°$. The factor $1-2sin\theta$ is equal to zero when $sin\theta = \frac{1}{2}$; that is, $sin^{-1}(\frac{1}{2}) = 30°$ and, since $sin(180-\theta) = sin\theta$ another solution is $180-30 = 150°$.

(c) $tan^2\theta - 2tan\theta = 1$

This equation can be treated as a quadratic equation where the variable, say, x, is equal to $tan\theta$. By making the x substitution and setting it equal to zero we have,

$x^2-2x-1=0$ then, using the quadratic formula: $x_{1,2} = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ where $a=1$, $b=-2$ and $c=-1$ the solutions are: $x_1 = 1+\sqrt{2}$ and $x_2 = 1-\sqrt{2}$. Since we define $x = tan\theta$, thus $tan^{-1}(1-\sqrt{2}) = -22.5°$ This angle is coterminal with $337.5°$ which is a solution in the given interval $(-22.5+360 = 337.5)$. Another solution takes into consideration the periodic property of the tangent function: $tan(180+\theta) = tan\theta$, therefore $180+(-22.5) = 157.5°$. The root $tan\theta = 1+\sqrt{2}$, leads to $tan^{-1}(1+\sqrt{2}) = 67.5°$ and, again by the periodic property of the tangent function:$180+67.5 = 247.5°$.

(d) $sin2\theta - \sqrt{3}cos2\theta = 0$ since we are dealing with 2θ the given interval, $0 \le \theta \le 360$ becomes $0 \le \theta \le 720$. Then, rearranging the equation, we have:

$$sin2\theta = \sqrt{3}cos2\theta$$
$$\frac{sin2\theta}{cos2\theta} = \sqrt{3}$$
$$tan2\theta = \sqrt{3}$$
$$tan^{-1}(\sqrt{3}) = 60°$$

Two other solutions for 2θ result from the periodic property of tangent, $180+60 = 240°$ and from adding another $360°$ —we are working in the interval $0 \le \theta \le 720$ —: $60+360 = 420°$ and, $240+360 = 600°$. That is, we have four solutions for 2θ, namely: $60°$, $240°$, $420°$, and $600°$; therefore, the corresponding values of $\theta = \dfrac{1}{2}(2\theta)$ are: $30°$, $120°$, $210°$, and $300°$.

13.

(a) $t(x) = tan3x$ $tan\theta$ has a period of $180°$; in this case, $\theta = 3x°$; therefore, the period, $T = \dfrac{180}{3} = 60°$.

(b) Given $tan3x = \dfrac{1}{2}$ therefore, a solution is $tan^{-1}(\frac{1}{2}) = 26.6°$ another solution is given by $tan(180+\theta) = tan\theta$ that is: $26.6+180 = 206.6°$ and, the third solution results from adding $360°$: $26.6°+360° = 386.6°$; now, solving for x (we have found solutions for $3x$) we have: $x = \dfrac{1}{3}(26.6) = 8.9°$; $x = \dfrac{1}{3}(206.6) = 68.9°$; and $x = \dfrac{1}{3}(386.6) = 128.9°$.

(c) $tan3x = -\dfrac{1}{2}$; $tan^{-1}(-\frac{1}{2}) = -26.6°$ which is not a solution in the interval under consideration; but, since $tan(180+\theta) = tan\theta$ we have $180-26.6 = 153.4°$ as a solution for $3x$; the x values is $x = \dfrac{1}{3}(153.4) = 51.1°$.

(d) $tan3x = 2$; therefore, $tan^{-1}(2) = 3x = 63.4°$. The x value is given by $x = \dfrac{1}{3}(63.4) = 21.1°$.

14.

(a) We are looking for a function, sine or cosine function, of the type: $y = A + Bsinkt$. Since the period is 24 hours $k = \dfrac{360}{24} = 15$; the maximum occurs when $sin15t = 1$; the minimum, when $sin15t = -1$; then we set up a simple system of two equations:

$$3.6 = A - B \quad \text{and} \quad 6 = A + B$$

If we add these equations together, the terms containing B add up to zero; we obtain:

$$9.6 = 2A \quad \text{Hence,} \quad A = 4.8 \ \text{ and,} \quad B = 6 - 4.8 = 1.2$$

Our equation is: $y = 4.8 + 1.2sinkt$
Note: similarly, we can model the phenomena using the cosine function. Or, instead of $y = A + Bsinkt$ we may use $y = A - Bsinkt$, considering that the maximum value is reached when $sinkt = -1$ and the minimum value when $sinkt = 1$, etc.

(b) Using the same approach as in (a): $k = \dfrac{360}{10} = 36$; using he equations of the form $y = A + Bsin36t$ our system of two equation is,

$$15000 = A - B \ \text{ and } \quad 28000 = A + B$$

$$43000 = 2A \quad \text{That is,} \quad A = 21500 \ \text{ and,} \quad B = 28000 - 21500 = 6500$$

$$y = 21500 + 6500sin36t$$

(c) $k = \dfrac{360}{360} = 1$; $y = A + Bsint$ The system of two equations:

$$2 = A - B \ \text{ and } \quad 22 = A + B$$

$$24 = 2A \quad \text{therefore} \quad A = 12, \quad B = 10$$

Our equation becomes,

$$y = 12 + 10sint$$

15.

The displacement, y centimeters, of the tip of one of the prongs from its rest position after t seconds is given by $y = 0.1sin(100000t)^\circ$

(a) The greatest displacement is 0.1 cm. It occurs when sine reaches its maximum value, one; that is, $sin(100000t)^\circ = 1$; by taking the inverse sine of 1, we find at which time it occurs: $sin^{-1}(1) = 90^\circ$; therefore, $100000t = 90$, and $t = \dfrac{90}{100000} = 0.0009 \ seconds$.

(b) The time for one complete oscillation is given by the period of the cosine function; thus, period, T,

$$T = \frac{360}{100000} = 3.6 \times 10^{-3} \ seconds.$$

(c) The number of completes oscillations per seconds is the frequency of the wave, that is, the reciprocal of the period (time for one complete oscillation): $Frequency = \dfrac{1}{3.6 \times 10^{-3}} = 277.77 \approx 278$, oscillations per second.

(d) See the sketch of the function's graph below. The total time during the first complete oscillation for which the tip is more than 0.06 cm from its rest position, is given by the time elapsed between t_1 and t_2 plus the time elapsed between t_3 and t_4. To find t_1 and t_2 we solve $0.06 = 0.1sin(100000t)^\circ$ which reduces to,

$$0.6 = sin(100000t)^\circ$$

$$sin^{-1}(0.6) = 36.87^\circ$$

$$t_1 = \frac{36.87}{100000} = 0.003687 \ s$$

Also, since $sin(180 - \theta) = sin\theta$, the solution in the II quadrant is: $180 - 36.87 = 143.13^\circ$. And t_2 is given by:

$$t_2 = \frac{143.13}{100000} = 0.0014313 \ s$$

The oscillation in the opposite direction —the prong oscillates from rest in one direction; and, from rest in the opposite direction, -0.06, as indicated in our sketch. Therefore,

$$-0.06 = 0.1sin(100000t)^{\circ}$$
$$-0.6 = sin(100000t)^{\circ}$$
$$sin^{-1}(-0.6) = -36.87^{\circ}$$

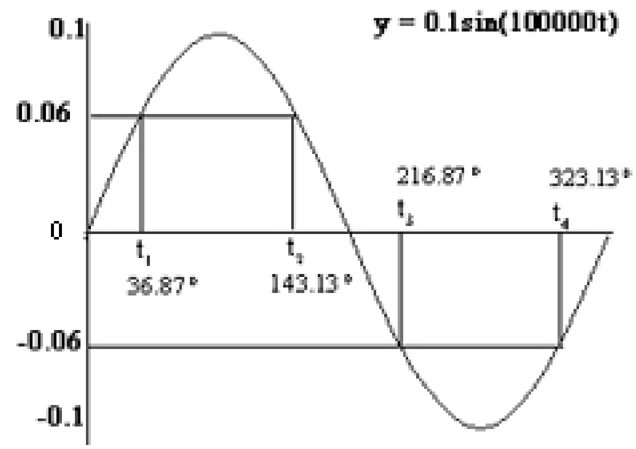
Again, since $sin(180-\theta) = sin\theta$, the solution in the III quadrant is: $180-(-36.87) = 216.87^{\circ}$. And t_3 is given by:

$$t_3 = \frac{216.87}{100000} = 0.0021687\ s$$

Notice, by analyzing the graph, that the prong vibrates beyond -0.06 until it reaches time t_4, in the IV quadrant. That angle is given by $360-36.87 = 323.13^{\circ}$. Thus,

$$t_4 = \frac{323.13}{100000} = 0.0032313\ s$$

The total time tip is more than 0.06 cm from its rest position= $(t_2-t_1)+(t_4-t_3) = (0.0014313-0.0003687)+(0.0032313-0.0021687) = 0.0021252 \approx 0.00213\ s$.



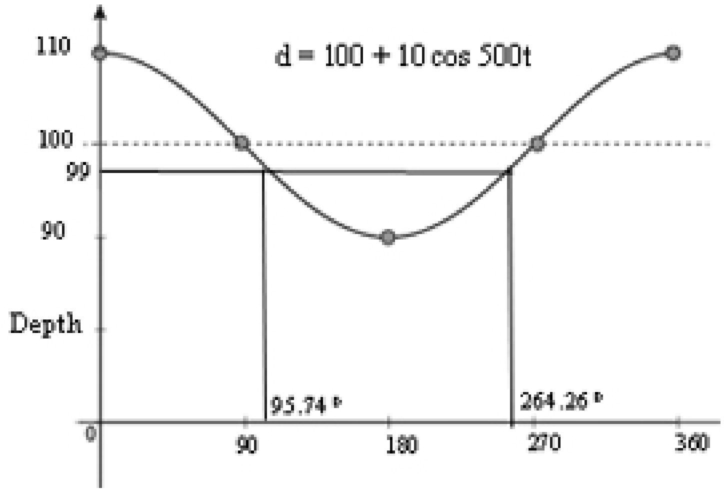
16.

(a) $d = 100 + 10\ cos500t^{\circ}$ The greatest depth occurs when $cos500t^{\circ} = 1$, $y = 100+10(1) = 110\ cm$; the least, when $cos500t^{\circ} = -1$, $y = 100+10(-1) = 90\ cm$.

(b) The lowest depth is the highest position (90cm), it occurs $cos500t^{\circ} = -1$ that is $cos^{-1}(-1) = 180$; therefore, $t = \frac{180}{500} = 0.36\ secs$.

(c) Time for a complete oscillation, that is, the period, is given by $T = \frac{360}{500} = 0.72\ secs$.

(d) The depth of the ball is less than 99 cm during the time elapsed between t_1 and t_2; t_1 is given by, $99 = 100+10cos500t^{\circ}$ which simplifies to $-1 = 10cos500t^{\circ}$; or, $-0.1 = cos500t^{\circ}$; then, solving for t, $cos^{-1}(-0.1) = 95.74^{\circ}$; therefore, $t = \frac{95.74}{500} = 0.1915\ secs$. The ball reaches $d = 99\ cm$ position again, at the instant when the angle is equal to 264.26°. Consider that $cos\theta = cos(-\theta)$; then, $cos\ 95.74 = cos(-95.74)$ and $-95.74+360 = 264.26^{\circ}$. At that point, $t = \frac{264.26}{500} = 0.5885\ secs$. Given that the period —see item (c)— is 0.72 $secs$ the proportion of time during a complete oscillation for which the depth of the ball is less than 99 cm, is given by $\frac{t_2-t_1}{0.72} = \frac{0.5885-0.1915}{0.72} = 0.4680$. The graph below depicts one complete oscillation, and the corresponding angles:



17.

The given equation of a particle that displacement y meters is: $y = a\ sin(kt + \alpha)°$; then,

(a) k in terms of T (period, or time for a complete oscillation) is given by $k = \frac{360}{T}$;

(b) The number, in terms of k of complete oscillations per second is the frequency, that is, the reciprocal of the period: $= \frac{T}{360}$;

18.

The given function is: $P = N - C\ coswt°$;

(a) T, the period, is known to be 50 weeks, then $w = \frac{360}{T} = \frac{360}{50} = 7.2$.

(b) (i) At the start of the year, $t = 0$, the number of birds is given by: $P = N - C\ cos7.2(0)° = N - C$.

(ii) The maximum number of birds: it occurs when $cos7.2t° = -1$; that is, $cos^{-1}(-1) = 180$; therefore, $7.2t = 180$, or $t = \frac{180}{7.2} = 25$. The maximum number of birds occurs at mid year, week 25.

19.

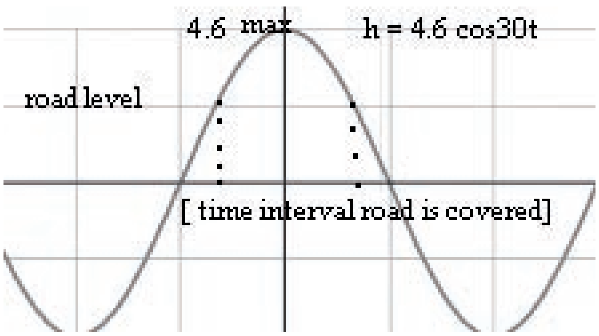
(a) The fact that *high tides occur every 12 hours* tells us that the period of the function is 12; therefore, since the period T is given by $T = \frac{360}{k}$, then $k = \frac{360}{12} = 30$.
(b) Our function becomes $h = 4.6cos30t$. Let's analyze its graph (see below): the maximum height is reached at $t = 0$ and it is equal to 4.6 m. The dots indicate the time interval the road is covered; since the cosine function is symmetrical about the y-axis the road level is reached when the water is rising, then the water level reaches its maximum and starts falling. The 3 hours the road is closed means that the water reaches the road 1.5 hr before its maximum of 4.6 m, and stays above the road level during the second 1.5 hr. In the function, time is set to be *time in hours from high tide*; therefore, the road level is reached 1.5 hrs before the tide reaches its maximum, $t = 1.5$; substituting,

$$h = 4.6cos30(1.5) = 3.25\ m$$

(c) Because of the road repair, the road is now covered for 2 hours 40 minutes; that is, the road level is reached 1 hour and 20 min from the tide maximum height. One hr and 20 mins = $1\frac{1}{3}$ hr or $\frac{4}{3}$hr; thus,

$$h = 4.6cos30\left(\frac{4}{3}\right) = 3.52\ m$$

The road level has been raised by $3.52 - 3.25 = 0.27m$ or $27cm$.



20.

The given equation is $h = Acos\alpha t + Bcos\beta t$; where the term $Acos\alpha$ is due to the sub's effect, whose period is 360 days; therefore, $\alpha = \frac{360}{360} = 1$; while the term $Bcos\beta t$ is due to the moon, whose period is 30 days; then, $\beta = \frac{360}{30} = 12$. The equation becomes $h = Acost + Bcos12t$. In determining the values of A and B, we take into consideration that $h = 5$ when $t = 0$; this fact gives us the following relationship between A and B:

$$5 = Acos(0) + Bcos12(0)$$

since $cos(0) = 1$, it reduces to:

$$5 = A + B$$

We know that *the magnitude of the attraction of the moon is assumed to be nine times the magnitude of the attraction of the sun*; that information is translated into the equation $9A = B$ (A is a factor in the sun's term; B a factor in the moon's term). Now we have a simple system of two equations, namely:

$$5 = A + B \quad \text{and} \quad 9A = B$$

Substituting the second equation into the first, we get:

$$5 = A + 9A \quad \text{or} \quad 5 = 10A \quad \text{that is} \quad A = 0.5$$

And, since $B = 9A$, it follows that $B = 9(0.5) = 4.5$.