6 Find the first three terms in the expansion in ascending powers of \( x \) of the following.
(a) \( (1 + x)^{22} \)  
(b) \( (1 - x)^{30} \)  
(c) \( (1 - 4x)^{18} \)  
(d) \( (1 + 6x)^{19} \)

7 Find the first three terms in the expansion, in ascending powers of \( x \), of \( (1 + 2x)^8 \). By substituting \( x = 0.01 \), find an approximation to \( 1.028 \).

8 Find the first three terms in the expansion, in ascending powers of \( x \), of \( (2 + 5x)^{12} \). By substituting a suitable value for \( x \), find an approximation to \( 2.005^{12} \) to 2 decimal places.

9 Expand \( (1 + 2x)^{16} \) up to and including the term in \( x^3 \). Deduce the coefficient of \( x^3 \) in the expansion of \( (1 + 3x)(1 + 2x)^{16} \).

10 Expand \( (1 - 3x)^{10} \) up to and including the term in \( x^2 \). Deduce the coefficient of \( x^2 \) in the expansion of \( (1 + 3x)^2(1 - 3x)^{10} \).

11 Given that the coefficient of \( x \) in the expansion of \( (1 + ax)(1 + 5x)^{40} \) is 207, determine the value of \( a \).

12 Simplify \( (1 - x)^8 + (1 + x)^8 \). Substitute a suitable value of \( x \) to find the exact value of \( 0.998 + 1.018 \).

13 Given that the expansion of \( (1 + ax)^n \) begins \( 1 + 36x + 576x^2 \), find the values of \( a \) and \( n \).

**Miscellaneous exercise 9**

1 Expand \( (3 + 4x)^3 \).

2 Find the first three terms in the expansions, in ascending powers of \( x \), of
(a) \( (1 + 4x)^{10} \)  
(b) \( (1 - 2x)^{16} \).

3 Find the coefficient of \( a^3b^5 \) in the expansions of
(a) \( (3a - 2b)^8 \)  
(b) \( (5a + \frac{1}{2}b)^8 \).

4 Expand \( (3 + 5x)^7 \) in ascending powers of \( x \) up to and including the term in \( x^2 \). By putting \( x = 0.01 \), find an approximation, correct to the nearest whole number, to \( 3.057 \).

5 Obtain the first four terms in the expansion of \( (2 + \frac{1}{4}x)^8 \) in ascending powers of \( x \). By substituting an appropriate value of \( x \) into this expansion, find the value of \( 2.0025^8 \) correct to three decimal places.

6 Find, in ascending powers of \( x \), the first three terms in the expansion of \( (2 - 3x)^8 \). Use the expansion to find the value of \( 1.997^8 \) to the nearest whole number.

7 Expand \( \left( x^2 + \frac{1}{x} \right)^3 \), simplifying each of the terms.

8 Expand \( \left( 2x - \frac{3}{x^2} \right)^4 \).
9 Expand and simplify \((x + \frac{1}{2x})^6 + (x - \frac{1}{2x})^6\).

10 Find the coefficient of \(x^2\) in the expansion of \(\left( x^4 + \frac{4}{x} \right)^3 \).

11 Find the term independent of \(x\) in the expansion of \(\left( 2x + \frac{5}{x} \right)^6 \).

12 Find the coefficient of \(y^4\) in the expansion of \((1 + y)^{12}\). Deduce the coefficient of 
   (a) \(y^4\) in the expansion of \((1 + 3y)^{12}\),
   (b) \(y^8\) in the expansion of \((1 - 2y^2)^{12}\),
   (c) \(x^8y^4\) in the expansion of \((x + \frac{1}{2}y)^{12}\).

13 Determine the coefficient of \(p^4q^7\) in the expansion of \((2p - q)(p + q)^{10}\).

14 Find the first three terms in the expansion of \((1 + 2x)^{20}\). By substitution of a suitable value 
   of \(x\) in each case, find approximations to
   (a) \(1.002^{20}\),
   (b) \(0.996^{20}\).

15 Write down the first three terms in the binomial expansion of \(\left( 2 - \frac{1}{2x^2} \right)^{10}\) in ascending 
   powers of \(x\). Hence find the value of \(1.995^{10}\) correct to three significant figures.

16 Two of the following expansions are correct and two are incorrect. Find the two expansions 
   which are incorrect.
   A: \((3 + 4x)^3 = 243 + 1620x + 4320x^2 + 5760x^3 + 3840x^4 + 1024x^5\)
   B: \((1 - 2x + 3x^2)^3 = 1 + 6x - 3x^2 + 28x^3 - 9x^4 + 54x^5 - 27x^6\)
   C: \((1-x)(1+4x)^4 = 1 + 15x + 80x^2 + 160x^3 - 256x^5\)
   D: \((2x + y)^2(3x + y)^2 = 108x^5 + 216x^4y + 171x^3y^2 + 67x^2y^3 + 13xy^4 + y^6\)

17 Find and simplify the term independent of \(x\) in the expansion of \(\left( \frac{x}{2x} + x^3 \right)^8\).

18 Find the term independent of \(x\) in the expansion of \(\left( 2x + \frac{1}{x^2} \right)^9\).

19 Evaluate the term which is independent of \(x\) in the expansion of \(\left( x^2 - \frac{1}{2x^2} \right)^{16}\)

20 Find the coefficient of \(x^{-12}\) in the expansion of \(\left( x^3 - \frac{1}{x} \right)^{24}\).

21 Expand \((1 + 3x + 4x^2)^4\) in ascending powers of \(x\) as far as the term in \(x^2\). By substituting 
   a suitable of \(x\), find an approximation to \(1.0304^4\).

22 Expand and simplify \((3x + 5)^3 - (3x - 5)^3\).
   Hence solve the equation \((3x + 5)^3 - (3x - 5)^3 = 730\).

23 Solve the equation \((7 - 6x)^3 + (7 + 6x)^3 = 1736\).
24 Find, in ascending powers of \( t \), the first three terms in the expansions of
(a) \((1 + \alpha t)^5\), \hspace{1cm} (b) \((1 - \beta t)^8\).

Hence find, in terms of \( \alpha \) and \( \beta \), the coefficient of \( t^2 \) in the expansion of
\((1 + \alpha t)^2(1 - \beta t)^8\).

25 (a) Show that
\[
\begin{align*}
(i) \quad \binom{6}{4} &= \binom{6}{2}, \\
(ii) \quad \binom{10}{3} &= \binom{10}{7}, \\
(iii) \quad \binom{15}{12} &= \binom{15}{3}, \\
(iv) \quad \binom{13}{6} &= \binom{13}{7}.
\end{align*}
\]
(b) State the possible values of \( x \) in each of the following.
\[
\begin{align*}
(i) \quad \binom{11}{4} &= \binom{11}{x}, \\
(ii) \quad \binom{16}{3} &= \binom{16}{x}, \\
(iii) \quad \binom{20}{7} &= \binom{20}{x}, \\
(iv) \quad \binom{45}{17} &= \binom{45}{x}.
\end{align*}
\]
(c) Use the definition \( \binom{n}{r} = \frac{n!}{r!(n-r)!} \) to prove that \( \binom{n}{r} = \binom{n}{n-r} \).

26 The inductive property \( \binom{n}{r+1} = \frac{n-r}{r+1} \binom{n}{r} \) was given in Section 8.4. Use this to prove the Pascal triangle property that \( \binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1} \).

27 (a) Show that
\[
\begin{align*}
(i) \quad 4 \times \binom{6}{2} &= 3 \times \binom{6}{3} = 6 \times \binom{5}{2}, \\
(ii) \quad 3 \times \binom{7}{4} &= 5 \times \binom{7}{5} = 7 \times \binom{6}{4}.
\end{align*}
\]
(b) State numbers \( a, b \) and \( c \) such that
\[
\begin{align*}
(i) \quad a \times \binom{8}{5} &= b \times \binom{8}{6} = c \times \binom{7}{5}, \\
(ii) \quad a \times \binom{9}{3} &= b \times \binom{9}{4} = c \times \binom{8}{3}.
\end{align*}
\]
(c) Prove that \( (n-r) \times \binom{n}{r} = (r+1) \times \binom{n}{r+1} = n \times \binom{n-1}{r} \).

28 Prove that \( \binom{n}{r-1} + 2 \binom{n}{r} + \binom{n}{r+1} = \binom{n+2}{r+1} \).

29 Find the value of \( 1.0003^{18} \) correct to 15 decimal places.

30 (a) Expand \((2\sqrt{2} + \sqrt{3})^4\) in the form \(a + b\sqrt{6}\), where \( a \) and \( b \) are integers.
(b) Find the exact value of \((2\sqrt{2} + \sqrt{3})^3\).

31 (a) Expand and simplify \((\sqrt{7} + \sqrt{5})^4 + (\sqrt{7} - \sqrt{5})^4\). By using the fact that \(0 < \sqrt{7} - \sqrt{5} < 1\), state the consecutive integers between which \((\sqrt{7} + \sqrt{5})^4\) lies.
(b) Without using a calculator, find the consecutive integers between which the value of \((\sqrt{3} + \sqrt{2})^8\) lies.

32 Find an expression, in terms of \( n \), for the coefficient of \( x \) in the expansion \((1 + 4x) + (1 + 4x)^2 + (1 + 4x)^3 + \ldots + (1 + 4x)^n\).

33 Given that \(a + b(1+x)^3 + c(1+2x)^3 + d(1+3x)^3 = x^3\)
for all values of \( x \), find the values of the constants \( a, b, c \) and \( d \).