Practice 10a, Questions

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Exercises 7 C —from number 6 to 16—, page 110 of Pure Mathematics 1 by Hugh Neil and Douglas Qualing.

6. At a speed of S km per hour a car will travel y kilometres on each litre of petrol, where $y = 5 + \frac{1}{5}S - \frac{1}{800}S^2$.
Calculate the speed at which the car should be driven for maximum economy.

7. A ball is thrown vertically upwards. At time t seconds its height h metres is given by $h = 20t - 5t^2$. Calculate the ball's maximum height above the ground.

8 The sum of two real numbers x and y is 12. Find the maximum value of their product xy.

9 The product of two positive real numbers x and y is 20 . Find the minimum possible value of their sum.

10 The volume of a cylinder is given by the formula $V = \pi r^2 h$. Find the greatest and least values of V if $r + h = 6$.

11 A loop of string of length 1 metre is formed into a rectangle with one pair of opposite sides each xcm. Calculate the value of x which will maximise the area enclosed by the string.

12 One side of a rectangular sheep pen is formed by a hedge. The other three sides are made using fencing. The length of the rectangle is x metres; 120 metres of fencing is available.
(a) Show that the area of the rectangle is $\frac{1}{2}x(120 - x)m^2$.
(b) Calculate the maximum possible area of the sheep pen.

13 A rectangular sheet of metal measures 50 cm by 40 cm . Equal squares of side x cm are cut from each corner and discarded. The sheet is then folded up to make a tray of depth x cm . What is the domain of possible values of x? Find the value of x which maximizes the capacity of the tray.

14 An open rectangular box is to be made with a square base, and its capacity is to be $4000cm^3$. Find the length of the side of the base when the amount of material used to make the box is as small as possible. (Ignore 'flaps'.)

15 An open cylindrical wastepaper bin, of radius r cm and capacity Vcm^3 is to have a surface area of $5000cm^2$.

(a) Show that $V = \frac{1}{2}r(5000 - \pi r^2)$

(b) Calculate the maximum possible capacity of the bin.

16 A circular cylinder is to fit inside a sphere of radius 10 cm . Calculate the maximum possible volume of the cylinder. (It is probably best to take as your independent variable the height, or half the height, of the cylinder.)