4 In each case the region enclosed between the following curves and the \( x \)-axis is rotated through \( 360^\circ \) about the \( x \)-axis. Find the volume of the solid generated.

(a) \( y=(x+1)(x-3) \)          (b) \( y=1-x^2 \)
(c) \( y=x^2-5x+6 \)           (d) \( y=x^2-3x \)

5 The region enclosed between the graphs of \( y=x \) and \( y=x^2 \) is denoted by \( R \). Find the volume generated when \( R \) is rotated through \( 360^\circ \) about

(a) the \( x \)-axis,          (b) the \( y \)-axis.

6 The region enclosed between the graphs of \( y=4x \) and \( y=x^2 \) is denoted by \( R \). Find the volume generated when \( R \) is rotated through \( 360^\circ \) about

(a) the \( x \)-axis,          (b) the \( y \)-axis.

7 The region enclosed between the graphs of \( y=\sqrt{x} \) and \( y=x^2 \) is denoted by \( R \). Find the volume generated when \( R \) is rotated through \( 360^\circ \) about

(a) the \( x \)-axis,          (b) the \( y \)-axis.

8 A glass bowl is formed by rotating about the \( y \)-axis the region between the graphs of \( y=x^2 \) and \( y=x^3 \). Find the volume of glass in the bowl.

9 The region enclosed by both axes, the line \( x=2 \) and the curve \( y=\frac{1}{8}x^2+2 \) is rotated about the \( y \)-axis to form a solid. Find the volume of this solid.

**Miscellaneous exercise 17**

1 The region bounded by the curve \( y=x^2+1 \), the \( x \)-axis, the \( y \)-axis and the line \( x=2 \) is rotated completely about the \( x \)-axis. Find, in terms of \( \pi \), the volume of the solid formed.
2 Explain why the coordinates \((x, y)\) of any point on a circle, centre \(O\), radius \(a\) satisfy the equation \(x^2 + y^2 = a^2\).

The semicircle above the \(x\)-axis is rotated about the \(x\)-axis through \(360^\circ\) to form a sphere of radius \(a\). Explain why the volume \(V\) of this sphere is given by

\[
V = 2\pi \int_0^a \left(a^2 - x^2\right) dx.
\]

Hence show that \(V = \frac{4}{3}\pi a^3\).

3 The ellipse with equation \(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\), shown in the diagram, has semi-axes \(a\) and \(b\).

The ellipse is rotated about the \(x\)-axis to form an ellipsoid. Find the volume of this ellipsoid.

Deduce the volume of the ellipsoid formed if, instead, the ellipse had been rotated about the \(y\)-axis.

4 The diagram shows the curve \(y = x^{-\frac{3}{2}}\).

(a) Show that the shaded area \(A\) is infinite.

(b) Find the shaded area \(B\).

(c) Area \(A\) is rotated through \(360^\circ\) about the \(x\)-axis. Find the volume generated.

(d) Area \(B\) is rotated through \(360^\circ\) about the \(y\)-axis. Find the volume generated.

5 Investigate the equivalent areas and volumes to those in Question 4 for the equations

(a) \(y = x^{-\frac{3}{2}}\),

(b) \(y = x^{-\frac{1}{4}}\).

6 Sketch the curve \(y = 9 - x^2\), stating the coordinates of the turning point and of the intersections with the axes.

The finite region bounded by the curve and the \(x\)-axis is denoted by \(R\).

(a) Find the area of \(R\) and hence or otherwise find \(\int_0^9 \sqrt{9 - y} dy\).

(b) Find the volume of the solid of revolution obtained when \(R\) is rotated through \(360^\circ\) about the \(x\)-axis.

(c) Find the volume of the solid of revolution obtained when \(R\) is rotated through \(360^\circ\) about the \(y\)-axis.

7 The region \(R\) is bounded by the part of the curve \(y = (x - 2)^{\frac{3}{2}}\) for which \(2 \leq x \leq 4\), the \(x\)-axis, and the line \(x = 4\). Find, in terms of \(\pi\), the volume of the solid obtained when \(R\) is rotated through four right angles about the \(x\)-axis.