3. Evaluate \( \int_{0}^{\frac{8}{3}} (3x - 2)^3 \, dx \).

4. Find \( \int_{0}^{4} \sqrt{2x + 1} \, dx \).

5. (a) Find \( \int \left( \frac{1}{x^3} + x^3 \right) \, dx \). (b) Evaluate \( \int_{0}^{\frac{8}{3}} \frac{1}{\sqrt[3]{x}} \, dx \).

6. Find the area of the region enclosed between the curve \( y = 12x^2 + 30x \) and the \( x \)-axis.

7. Given that \( \int_{-a}^{a} 15x^2 \, dx = 3430 \), find the value of the constant \( a \).

8. The diagram shows the curve \( y = x^3 \). The point \( P \) has coordinates \((3, 27)\) and \( PQ \) is the tangent to the curve at \( P \). Find the area of the region enclosed between the curve, \( PQ \) and the \( x \)-axis.

9. The diagram shows the curve \( y = (x - 2)^2 + 1 \) with minimum point \( P \). The point \( Q \) on the curve is such that the gradient of \( PQ \) is \( 2 \). Find the area of the region, shaded in the diagram, between \( PQ \) and the curve.

10. Evaluate \( \int_{0}^{2} x(x - 1)(x - 2) \, dx \) and explain your answer with reference to the graph of \( y = x(x - 1)(x - 2) \).

11. (a) Find \( \int x(x^2 - 2) \, dx \).

   (b) The diagram shows the graph of \( y = x(x^2 - 2) \) for \( x \geq 0 \). The value of \( a \) is such that the two shaded regions have equal areas. Find the value of \( a \).

12. Given that \( \int_{1}^{p} (8x^3 + 6x) \, dx = 39 \), find two possible values of \( p \). Use a graph to explain why there are two values.

13. Show that the area enclosed between the curves \( y = 9 - x^2 \) and \( y = x^2 - 7 \) is \( \frac{128\sqrt{2}}{3} \).
14 The diagram shows a sketch of the graph of \( y = x^2 \) and the normal to the curve at the point \( A(1,1) \).

(a) Use differentiation to find the equation of the normal at \( A \). Verify that the point \( B \) where the normal cuts the curve again has coordinates \( \left(-\frac{3}{2}, \frac{9}{4}\right) \).

(b) The region which is bounded by the curve and the normal is shaded in the diagram. Calculate its area, giving your answer as an exact fraction.

15 Given that \( f(x) \) and \( g(x) \) are two functions such that \( \int_{0}^{4} f(x)\,dx = 17 \) and \( \int_{0}^{4} g(x)\,dx = 11 \), find, where possible, the value of each of the following.

(a) \( \int_{0}^{4} (f(x) - g(x))\,dx \)  
(b) \( \int_{0}^{4} (2f(x) + 3g(x))\,dx \)

(c) \( \int_{0}^{2} f(x)\,dx \)  
(d) \( \int_{0}^{4} (f(x) + 2x + 3)\,dx \)

(e) \( \int_{0}^{1} f(x)\,dx + \int_{1}^{4} f(x)\,dx \)  
(f) \( \int_{4}^{0} g(x)\,dx \)

(g) \( \int_{1}^{5} f(x-1)\,dx \)  
(h) \( \int_{-4}^{0} g(-t)\,dt \)

16 The diagram shows the graph of \( y = \sqrt[3]{x} - x^2 \). Show by integration that the area of the region (shaded in the diagram) between the curve and the \( x \)-axis is \( \frac{5}{12} \).

17 The diagram shows a sketch of the graph of the curve \( y = x^3 - x \) together with the tangent to the curve at the point \( A(1,0) \).

(a) Use differentiation to find the equation of the tangent to the curve at \( A \), and verify that the point \( B \) where the tangent cuts the curve again has coordinates \( (-2,-6) \).

(b) Use integration to find the area of the region bounded by the curve and the tangent (shaded in the diagram), giving your answer as a fraction in its lowest terms.
18 The diagram shows part of the curve \( y = x^n \), where \( n > 1 \).

The point \( P \) on the curve has \( x \)-coordinate \( a \). Show that the curve divides the rectangle \( OAPB \) into two regions whose areas are in the ratio \( n:1 \).

19 Find the stationary points on the graph of \( y = x^4 - 8x^2 \). Use your answers to make a sketch of the graph. Show that the graphs of \( y = x^4 - 8x^2 \) and \( y = x^2 \) enclose two finite regions. Find the area of one of them.

20 Using the same axes, make sketches of the graphs of \( y = x^3 \) and \( y = (x + 1)^3 - 1 \). Then sketch on a larger scale the finite area enclosed between them.

Find the area of the region.

21 A function \( f(x) \) with domain \( x > 0 \) is defined by \( f(x) = 6x^4 - \frac{2}{x^3} \).

(a) Find the values of \( \int_2^3 f(x) \, dx \) and \( \int_2^\infty f(x) \, dx \).

(b) Find the coordinates of

(i) the point where the graph of \( y = f(x) \) crosses the \( x \)-axis,

(ii) the minimum point on the graph.

Use your answers to draw a sketch of the graph, and hence explain your answers to part (a).

22 The diagram shows the curve \( y = (2x - 3)^3 \).

(a) Find the \( x \)-coordinates of the two points on the curve which have gradient 6.

(b) The region shaded in the diagram is bounded by part of the curve and by the two axes. Find, by integration, the area of this region. (OCR)

23 The diagram shows the curve with equation \( y = \sqrt{4x + 1} \) and the normal to the curve at the point \( A \) with coordinates \((6, 5)\).

(a) Show that the equation of the normal to the curve at \( A \) is \( y = -\frac{5}{2}x + 20 \).

(b) Find the area of the region (shaded in the diagram) which is enclosed by the curve, the normal and the \( x \)-axis. Give your answer as a fraction in its lowest terms.