

Miscellaneous exercise 14

- 1 In a geometric progression, the fifth term is 100 and the seventh term is 400. Find the first term.
- 2 A geometric series has first term a and common ratio $\frac{1}{\sqrt{2}}$. Show that the sum to infinity of the series is $a(2 + \sqrt{2})$. (Hint: $(\sqrt{2} - 1)(\sqrt{2} + 1) = 1$.)
- 3 The n th term of a sequence is ar^{n-1} , where a and r are constants. The first term is 3 and the second term is $-\frac{3}{4}$. Find the values of a and r .
Hence find the sum of the first n terms of the sequence.
- 4 Evaluate, correct to the nearest whole number,
$$0.99 + 0.99^2 + 0.99^3 + \dots + 0.99^{99}.$$
- 5 Find the sum of the infinite series $\frac{1}{10^3} + \frac{1}{10^6} + \frac{1}{10^9} + \dots$, expressing your answer as a fraction in its lowest terms.
Hence express the infinite recurring decimal $0.108\ 108\ 108\dots$ as a fraction in its lowest terms.
- 6 A geometric series has first term 1 and common ratio r . Given that the sum to infinity of the series is 5, find the value of r .
Find the least value of n for which the sum of the first n terms of the series exceeds 4.9.
- 7 In a geometric series, the first term is 12 and the fourth term is $-\frac{3}{2}$. Find the sum, S_n , of the first n terms of the series.
Find the sum to infinity, S_∞ , of the series and the least value of n for which the magnitude of the difference between S_n and S_∞ is less than 0.001.
- 8 A geometric series has non-zero first term a and common ratio r , where $0 < r < 1$. Given that the sum of the first 8 terms of the series is equal to half the sum to infinity, find the value of r , correct to 3 decimal places. Given also that the 17th term of the series is 10, find a .

