Miscellaneous exercise 14

1 In a geometric progression, the fifth term is 100 and the seventh term is 400. Find the first term.

2 A geometric series has first term a and common ratio $\frac{1}{\sqrt{2}}$. Show that the sum to infinity of the series is $a\left(2+\sqrt{2}\right)$. (Hint: $\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)=1$.)

3 The nth term of a sequence is ar^{n-1}, where a and r are constants. The first term is 3 and the second term is $-\frac{3}{4}$. Find the values of a and r.

Hence find the sum of the first n terms of the sequence.

4 Evaluate, correct to the nearest whole number,

$$0.99+0.99^2+0.99^3+\ldots+0.99^{99}.$$

5 Find the sum of the infinite series $\frac{1}{10^3}+\frac{1}{10^6}+\frac{1}{10^9}+\ldots$, expressing your answer as a fraction in its lowest terms.

Hence express the infinite recurring decimal $0.108\,108\,108\ldots$ as a fraction in its lowest terms.

6 A geometric series has first term 1 and common ratio r. Given that the sum to infinity of the series is 5, find the value of r.

Find the least value of n for which the sum of the first n terms of the series exceeds 4.9.

7 In a geometric series, the first term is 12 and the fourth term is $-\frac{3}{2}$. Find the sum, S_n, of the first n terms of the series.

Find the sum to infinity, S_∞, of the series and the least value of n for which the magnitude of the difference between S_n and S_∞ is less than 0.001.

8 A geometric series has non-zero first term a and common ratio r, where $0<r<1$. Given that the sum of the first 8 terms of the series is equal to half the sum to infinity, find the value of r, correct to 3 decimal places. Given also that the 17th term of the series is 10, find a.