Miscellaneous exercise 1

1 Show that the triangle formed by the points \((-2,5), (1,3)\) and \((5,9)\) is right-angled.

2 Find the coordinates of the point where the lines \(2x + y = 3\) and \(3x + 5y - 1 = 0\) meet.

3 A triangle is formed by the points \(A(-1,3), B(5,7)\) and \(C(0,8)\).
   (a) Show that the angle \(ACB\) is a right angle.
   (b) Find the coordinates of the point where the line through \(B\) parallel to \(AC\) cuts the \(x\)-axis.

4 \((7,2)\) and \((1,4)\) are two vertices of a square \(ABCD\).
   (a) Find the equation of the diagonal \(BD\).
   (b) Find the coordinates of \(B\) and of \(D\).

5 A quadrilateral \(ABCD\) is formed by the points \((-3,2), (4,3), (9,-2)\) and \((2,-3)\).
   (a) Show that all four sides are equal in length.
   (b) Show that \(ABCD\) is not a square.

6 \(P\) is the point \((7,5)\) and \(l_1\) is the line with equation \(3x + 4y = 16\).
   (a) Find the equation of the line \(l_2\) which passes through \(P\) and is perpendicular to \(l_1\).
   (b) Find the point of intersection of the lines \(l_1\) and \(l_2\).
   (c) Find the perpendicular distance of \(P\) from the line \(l_1\).

7 Prove that the triangle with vertices \((-2,8), (3,20)\) and \((11,8)\) is isosceles. Find its area.

8 The three straight lines \(y = x,\ 7y = 2x\) and \(4x + y = 60\) form a triangle. Find the coordinates of its vertices.

9 Find the equation of the line through \((1,3)\) which is parallel to \(2x + 7y = 5\). Give your answer in the form \(ax + by = c\).

10 Find the equation of the perpendicular bisector of the line joining \((2,-5)\) and \((-4,3)\).

11 The points \((1,2), (3,5), (6,6)\) and \(D\) form a parallelogram. Find the coordinates of the mid-point of \(AC\). Use your answer to find the coordinates of \(D\).

12 The point \(P\) is the foot of the perpendicular from the point \((0,3)\) to the line \(y = 3x\).
   (a) Find the equation of the line \(AP\).
   (b) Find the coordinates of the point \(P\).
   (c) Find the perpendicular distance of \(A\) from the line \(y = 3x\).