Math 101-08
12–1–2008

Review Problems for the Final

These problems are provided to help you study. The presence of a problem on this handout does not imply that there *will* be a similar problem on the test. And the absence of a topic does not imply that it *won't* appear on the test.

1. Simplify the following expressions. Express all powers in terms of positive exponents.

(a) $\dfrac{-4 - |-5|}{|-5 - (-3)|}$.

(b) $(-2x^2y^{1/3})^6$.

(c) $\sqrt[3]{\dfrac{-8x^5y^{-8}}{x^{-1}y^4}}$.

(d) $\sqrt{338} + 5\sqrt{98}$.

(e) $\sqrt[3]{250x^5y^9}$.

2. Rationalize $\dfrac{3+5\sqrt{2}}{6-3\sqrt{2}}$.

3. Calvin Butterball has 29 coins, all of which are dimes or quarters. The value of the coins is \$5.45. How many dimes does he have?

4. How many pounds of chocolate truffles worth \$2.50 per pound must be mixed with 5 pounds of spackling compound worth \$1.70 per pound to yield a mixture worth \$2.00 per pound?

5. Calvin leaves the city and travels north at 23 miles per hour. Phoebe starts 2 hours later, travelling south at 17 miles per hour. Some time after Phoebe starts travelling, the distance between them is 166 miles. How many hours has Calvin been travelling at this instant?

6. Calvin can eat 160 double cheeseburgers in 4 hours, while Bonzo can eat 300 double cheeseburgers in 5 hours. If Calvin, Bonzo, and Phoebe work together, they can eat 510 double cheeseburgers in 3 hours. How many hours would Phoebe take to eat 280 double cheeseburgers if she eats alone?

7. The product of two numbers is 156. The second number is 11 more than twice the first. Both numbers are positive. What are the two numbers?

8. The hypotenuse of a right triangle is 1 more than 3 times the smallest side. The third side is 1 less than 3 times the smallest side. Find the lengths of the sides.

9. Solve the inequality:

(a) $x^2 - 7x + 10 < 0$.

(b) $\dfrac{x+4}{3-x} > 0$.

(c) $|2x - 7| > 3$.

10. (a) Write the expression as a single logarithm: $\ln 2 + 5\ln x^2 + \dfrac{1}{2}\ln y$.

(b) Write the expression as a single logarithm: $4\ln x - \dfrac{1}{3}\ln y - 7\ln z$.

11. (a) Write the expression as a sum or difference of logarithms: $\log_{10} \dfrac{x^3y^4}{\sqrt{z}}$.

(b) Write the expression as a sum or difference of logarithms: $\ln \sqrt{\dfrac{\sqrt{x+2}}{(y-1)^5}}$.

12. Solve for x, writing your answer in decimal form correct to 3 places: $3^{2x} = 7$.

13. Solve: $\sqrt{5-2x} - 1 = \sqrt{6-x}$.

14. Solve: $\dfrac{13}{x^2-4} = \dfrac{2}{x-2} - \dfrac{3}{x+2}$.

15. Simplify:

(a) $-(49^{3/2})$.

(b) $(-49)^{3/2}$.

(c) $(-64)^{-5/3}$.

(d) $\sqrt{360x^2y^3}$, assuming that the variables represent nonnegative quantities.

16. Find values for a and b which prove that the following statement is not an algebraic identity:

$$\text{“}\sqrt{a^2+b^2} \stackrel{?}{=} a+b\text{”}$$

17. (a) Combine the fractions over a common denominator: $\dfrac{1}{x^2-3x} + \dfrac{2x}{x^2-9}$.

(b) Combine the fractions over a common denominator: $\dfrac{y}{x^2-xy} - \dfrac{y}{x^2-y^2}$.

(c) Simplify: $\dfrac{\dfrac{x^2+1}{1-\dfrac{1}{x}} - \dfrac{x^2+1}{1+\dfrac{1}{x}}}{x+\dfrac{1}{x}}$.

18. Compute (without using a calculator):

(a) $\log_5 125$.

(b) $\log_5 5^{89}$.

(c) $\log_5 \dfrac{1}{25}$.

(d) $\log_{47} 1$.

(e) $\log_8 4$.

(f) $e^{\ln 42}$.

19. Find the inverse function of $f(x) = \dfrac{x}{x-1} + 1$.

20. Find the domain of the function $f(x) = \dfrac{1}{\sqrt{x^2-x-2}}$.

21. Solve for x:

(a) $|2x-5| = 3$.

(b) $3x - 5 = 2(x + 1)$.

(c) $4(x - 2) < x + 7$.

(d) $x^2 - 4x = 5$.

(e) $x^2 - 4x = -5$.

22. Simplify the expressions, writing each result in the form $a + bi$:

(a) i^{33}.

(b) $(3 - i)(4 + 5i)$.

(c) $\dfrac{1}{2 + 3i}$.

(d) $\dfrac{1 - 2i}{6 + 8i}$.

(e) $(2 + 3i)^2$.

(f) $\dfrac{3 - 2i}{4 + i}$.

23. Find the quotient and the remainder when $x^4 + 3x^3 - 2x^2 + 1$ is divided by $x^2 - 1$.

24. Solve the following equations, giving *exact* answers:

(a) $e^{2x} + 5 = 6e^x$.

(b) $3^{x+2} = 7^{2x+1}$.

(c) $4^{3x+1} = 5^x$.

(d) $\ln(x - 2) + \ln(x + 2) = \ln 3x$.

(e) $(\ln x)^2 - 3 \ln x - 4 = 0$.

25. (a) Simplify $\sqrt{500}$.

(b) Simplify $\sqrt{-300}$.

(c) Rationalize $\dfrac{2 + \sqrt{7}}{1 - 3\sqrt{7}}$.

26. Find the equation of the line:

(a) Which passes through the points $(2, 3)$ and $(-11, 1)$.

(b) Which passes through the point $(3, 4)$ and is perpendicular to the line $2x - 8y = 5$.

(c) Which is parallel to the line $3y - 6x + 5 = 0$ and has y-intercept -17.

27. Solve the system of equations for x and y:

$$2x + 5y = 7, \quad x + 3y = -4.$$

28. Suppose $\log_a x = -5$ and $\log_a y = 2$. Find:

(a) $\log_a \dfrac{1}{\sqrt[3]{x}}$.

(b) $\log_a(a^6y^2)$.

(c) $\log_a \dfrac{x^{-3}}{y^2}$.

29. (a) Simplify and write the result using positive exponents: $\dfrac{(-3x^2)^5y^{-7}}{-9x^3y^6}$.

(b) Simplify and write the result using positive exponents: $4(x^{1/3}y^{2/5})^2 \cdot (-3y^{-3/5})^2x^{-1/6}$.

30. (a) Simplify, cancelling any common factors: $\dfrac{\dfrac{x^2-2x}{x^2-4}}{\dfrac{x^3-3x^2}{x^2-x-6}}$.

(b) Simplify, cancelling any common factors: $\dfrac{\dfrac{a^3-2a^2b}{a^3-4ab^2}}{\dfrac{a^4+3a^3b}{a^2+ab-2b^2}}$.

(c) Simplify, cancelling any common factors: $\dfrac{\dfrac{x^2-5x}{x^3-4x^2}}{\dfrac{x^2-10x+25}{x^2-16}}$.

31. (a) Find the equation of the parabola which passes through the point $(3, 7)$ and has vertex $(2, 3)$.

(b) Write the quadratic function $f(x) = x^2 - 6x + 34$ in standard (vertex) form.

32. Find the center and the radius of the circle whose equation is

$$x^2 - 4x + y^2 + 6y = 12.$$

33. Suppose that \$1000 is invested at 6% annual interest, compounded monthly. How many years must pass before the account is worth at least \$2000? (Round up to the nearest year.)

Solutions to the Review Problems for the Final

1. Simplify the following expressions. Express all powers in terms of positive exponents.

(a) $\dfrac{-4-|-5|}{|-5-(-3)|}$.

$$\frac{-4-|-5|}{|-5-(-3)|} = \frac{-4-5}{|-5+3|} = \frac{-9}{|-2|} = -\frac{9}{2} \quad \square$$

(b) $(-2x^2y^{1/3})^6$.

$$(-2x^2y^{1/3})^6 = (-2)^6(x^2)^6(y^{1/3})^6 = 64x^{12}y^2 \quad \square$$

(c) $\sqrt[3]{\dfrac{-8x^5y^{-8}}{x^{-1}y^4}}$.

$$\sqrt[3]{\frac{-8x^5y^{-8}}{x^{-1}y^4}} = \left(\frac{-8x^5y^{-8}}{x^{-1}y^4}\right)^{1/3} = \left(-8x^6y^{-12}\right)^{1/3} = (-8)^{1/3}(x^6)^{1/3}(y^{-12})^{1/3} = -2x^2y^{-4} = \frac{-2x^2}{y^4} \quad \square$$

(d) $\sqrt{338} + 5\sqrt{98}$.

$$\sqrt{338} + 5\sqrt{98} = \sqrt{169}\sqrt{2} + 5\sqrt{49}\sqrt{2} = 13\sqrt{2} + 5 \cdot 7\sqrt{2} = 13\sqrt{2} + 35\sqrt{2} = 48\sqrt{2}. \quad \square$$

(e) $\sqrt[3]{250x^5y^9}$.

$$\sqrt[3]{250x^5y^9} = \sqrt[3]{250}\sqrt[3]{x^5}\sqrt[3]{y^9} = \sqrt[3]{125 \cdot 2}\sqrt[3]{x^3 \cdot x^2}\sqrt[3]{y^9} = \sqrt[3]{125}\sqrt[3]{2}\sqrt[3]{x^3}\sqrt[3]{x^2}\sqrt[3]{y^9} = 5xy^3\sqrt[3]{2x^2}. \quad \square$$

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2. Rationalize $\dfrac{3 + 5\sqrt{2}}{6 - 3\sqrt{2}}$.

$$\frac{3 + 5\sqrt{2}}{6 - 3\sqrt{2}} = \frac{3 + 5\sqrt{2}}{6 - 3\sqrt{2}} \cdot \frac{6 + 3\sqrt{2}}{6 + 3\sqrt{2}} = \frac{18 + 9\sqrt{2} + 30\sqrt{2} + 15(\sqrt{2})^2}{36 - 9(\sqrt{2})^2} = \frac{48 + 39\sqrt{2}}{18} = \frac{16 + 13\sqrt{2}}{6}. \quad \square$$

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3. Calvin Butterball has 29 coins, all of which are dimes or quarters. The value of the coins is \$5.45. How many dimes does he have?

Let d be the number of dimes and let q be the number of quarters.

	number	$\cdot$	value	$=$	total value
dimes	d	$\cdot$	10	$=$	$10d$
quarters	q	$\cdot$	25	$=$	$25q$
coins	29				545

The first column says $d + q = 29$. The last column says $10d + 25q = 545$. The first of these equations gives $d = 29 - q$. Substitute this into $10d + 25q = 545$ to get $10(29 - q) + 25q = 545$. Now solve for q:

$$10(29 - q) + 25q = 545, \quad 290 - 10q + 25q = 545, \quad 15q = 255, \quad q = 17.$$

There are 17 quarters and $d = 29 - 17 = 12$ dimes. $\square$

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4. How many pounds of chocolate truffles worth \$2.50 per pound must be mixed with 5 pounds of spackling compound worth \$1.70 per pound to yield a mixture worth \$2.00 per pound?

Let x be the number of pounds of truffles.

	pounds	$\cdot$	price per pound	$=$	value
spackling compound	5	$\cdot$	170	$=$	850
truffles	x	$\cdot$	250	$=$	$250x$
mixture	$(x + 5)$	$\cdot$	200	$=$	$850 + 250x$

The last line of the table gives

$$200(x + 5) = 850 + 250x.$$

Solve for x:

$$200x + 1000 = 850 + 250x, \quad 200x + 150 = 250x, \quad 150 = 50x, \quad x = 3.$$

The mixture needs 3 pounds of truffles. □

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5. Calvin leaves the city and travels north at 23 miles per hour. Phoebe starts 2 hours later, travelling south at 17 miles per hour. Some time after Phoebe starts travelling, the distance between them is 166 miles. How many hours has Calvin been travelling at this instant?

Let t be the time Calvin has travelled. Then Phoebe has travelled $t-2$ hours. Let x be the distance Calvin has travelled in this time. Then Phoebe has travelled $166-x$ miles.

	speed	time	distance
Calvin	23	t	x
Phoebe	17	$t-2$	$166-x$

The first line says $23t = x$. The second line says $17(t-2) = 166-x$. Plug $x = 23t$ into $17(t-2) = 166-x$ and solve for t:

$$\begin{aligned} 17(t-2) &= 166 - 23t \\ 17t - 34 &= 166 - 23t \\ 40t &= 200 \\ t &= 5 \end{aligned}$$

Calvin has been travelling for 5 hours. □

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6. Calvin can eat 160 double cheeseburgers in 4 hours, while Bonzo can eat 300 double cheeseburgers in 5 hours. If Calvin, Bonzo, and Phoebe work together, they can eat 510 double cheeseburgers in 3 hours. How many hours would Phoebe take to eat 280 double cheeseburgers if she eats alone?

Let x be the number of burgers Calvin can eat per hour. Let y be the number of burgers Bonzo can eat per hour. Let z be the number of burgers Phoebe can eat per hour.

	burgers per hour	$\cdot$	hours	$=$	burgers
Calvin	x	$\cdot$	4	$=$	160
Bonzo	y	$\cdot$	5	$=$	300
Phoebe	z	$\cdot$	t	$=$	280
together	$(x+y+z)$	$\cdot$	3	$=$	510

The first equation says $4x = 160$, so $x = 40$.
The second equation says $5y = 300$, so $y = 60$.
The last equation says $3(x+y+z) = 510$. Divide by 3: $x+y+z = 170$. Substitute $x = 40$ and $y = 60$:

$$40 + 60 + z = 170, \quad z = 70.$$

The third equation says $zt = 280$. Substitute $z = 70$: $70t = 280$, so $t = 4$ hours. □

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7. The product of two numbers is 156. The second number is 11 more than twice the first. Both numbers are positive. What are the two numbers?

Let x and y be the two numbers.
The product of two numbers is 156, so $xy = 156$.

The second number is 11 more than twice the first, so $y = 11 + 2x$.
Substitute $y = 11 + 2x$ into $xy = 156$:

$$x(11 + 2x) = 156, \quad 11x + 2x^2 = 156.$$

Then

$$\begin{array}{rrrrrrr} & 2x^2 & + & 11x & & = & 156 \\ - & & & & 156 & & 156 \\ \hline & 2x^2 & + & 11x & - & 156 & = & 0 \end{array}$$

Factor and solve:

$$\begin{gathered} 2x^2 - 11x - 156 = 0 \\ (2x - 13)(x + 12) = 0 \end{gathered}$$

$$\swarrow \qquad\qquad\qquad\qquad \searrow$$

$$\begin{array}{ccc} 2x - 13 = 0 & \qquad\qquad\qquad & x + 12 = 0 \\ x = \dfrac{13}{2} & & x = -12 \end{array}$$

$x = -12$ is ruled out, because x is supposed to be positive. Therefore, $x = \dfrac{13}{2}$, and $y = 11 + 2x = 24$.
□

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8. The hypotenuse of a right triangle is 1 more than 3 times the smallest side. The third side is 1 less than 3 times the smallest side. Find the lengths of the sides.

Let s be the length of the smallest side, let t be the length of the third side, and let h be the length of the hypotenuse. By Pythagoras' theorem,

$$h^2 = s^2 + t^2.$$

The hypotenuse of a right triangle is 1 more than 3 times the smallest side, so $h = 3s + 1$.
The third side is 1 less than 3 times the smallest side, so $t = 3s - 1$.
Plug $h = 3s + 1$ and $t = 3s - 1$ into $h^2 = s^2 + t^2$ and solve for s:

$$\begin{aligned} h^2 &= s^2 + t^2 \\ (3s + 1)^2 &= s^2 + (3s - 1)^2 \\ 9s^2 + 6s + 1 &= s^2 + 9s^2 - 6s + 1 \\ 0 &= s^2 - 12s \\ 0 &= s(s - 12) \end{aligned}$$

The possible solutions are $s = 0$ and $s = 12$. Now $s = 0$ is ruled out, since a triangle can't have a side of length 0. Therefore, $s = 12$ is the only solution. The other sides are

$$h = 3 \cdot 12 + 1 = 37 \quad \text{and} \quad t = 3 \cdot 12 - 1 = 35. \quad \square$$

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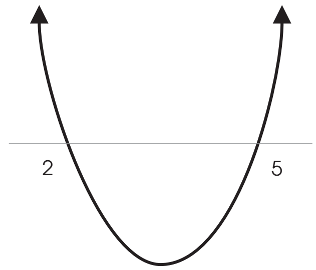
9. Solve the inequality:

(a) $x^2 - 7x + 10 < 0$.

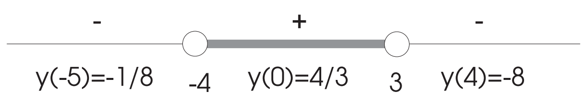
The graph of $y = x^2 - 7x + 10$ is a parabola opening upward. $x^2 - 7x + 10 = 0$ gives $(x-2)(x-5) = 0$, so the roots are $x = 2$ and $x = 5$.



The solution is $2 < x < 5$. ☐

(b) $\dfrac{x+4}{3-x} > 0$.

$y = \dfrac{x+4}{3-x}$ equals 0 when $x = -4$ and is undefined when $x = 3$. Set up a sign chart with these break points:



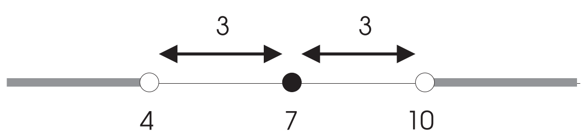
The solution is $-4 < x < 3$. ☐

(c) $|2x - 7| > 3$.

Translate the inequality:

$\lvert 2x-7 \rvert$	$>$	3
the distance from $2x$ to 7	is greater than	3

Draw the picture:



Since $2x$ is greater than 3 units from 7, I must have

$$2x < 4 \quad \text{or} \quad 2x > 10.$$

These inequalities give

$$x < 2 \quad \text{or} \quad x > 5. \quad \square$$

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10. (a) Write the expression as a single logarithm: $\ln 2 + 5 \ln x^2 + \dfrac{1}{2} \ln y$.

$$\ln 2 + 5\ln x^2 + \frac{1}{2}\ln y = \ln 2 + \ln(x^2)^5 + \ln y^{1/2} = \ln 2 + \ln x^{10} + \ln \sqrt{y} = \ln 2x^{10}\sqrt{y}. \quad \square$$

(b) Write the expression as a single logarithm: $4 \ln x - \dfrac{1}{3} \ln y - 7 \ln z$.

$$4\ln x - \frac{1}{3}\ln y - 7\ln z = \ln x^4 - \ln y^{1/3} - \ln z^7 = \ln\left(\frac{x^4}{y^{1/3}z^7}\right). \quad \square$$

11. (a) Write the expression as a sum or difference of logarithms: $\log_{10} \dfrac{x^3 y^4}{\sqrt{z}}$.

$$\log_{10} \frac{x^3 y^4}{\sqrt{z}} = \log_{10}(x^3 y^4) - \log_{10} \sqrt{z} = \log_{10} x^3 + \log_{10} y^4 - \log_{10} z^{1/2} = 3 \log_{10} x + 4 \log_{10} y - \frac{1}{2} \log_{10} z. \quad \square$$

(b) Write the expression as a sum or difference of logarithms: $\ln \sqrt{\dfrac{\sqrt{x+2}}{(y-1)^5}}$.

$$\ln \sqrt{\frac{\sqrt{x+2}}{(y-1)^5}} = \ln \left(\frac{\sqrt{x+2}}{(y-1)^5} \right)^{1/2} = \frac{1}{2} \ln \frac{\sqrt{x+2}}{(y-1)^5} = \frac{1}{2} \left(\ln \sqrt{x+2} - \ln(y-1)^5 \right) =$$

$$\frac{1}{2} \left(\ln(x+2)^{1/2} - \ln(y-1)^5 \right) = \frac{1}{2} \left(\frac{1}{2} \ln(x+2) - 5 \ln(y-1) \right) = \frac{1}{4} \ln(x+2) - \frac{5}{2} \ln(y-1). \quad \square$$

12. Solve for x, writing your answer in decimal form correct to 3 places: $3^{2x} = 7$.

$$\begin{aligned} 3^{2x} &= 7 \\ \ln 3^{2x} &= \ln 7 \\ (2x) \ln 3 &= \ln 7 \\ 2x &= \frac{\ln 7}{\ln 3} \\ x &= \frac{\ln 7}{2 \ln 3} \approx 0.88562 \quad \square \end{aligned}$$

13. Solve: $\sqrt{5-2x} - 1 = \sqrt{6-x}$.

Square both sides:

$$(\sqrt{5-2x} - 1)^2 = (\sqrt{6-x})^2, \quad (5-2x) - 2\sqrt{5-2x} + 1 = 6 - x, 6 - 2x - 2\sqrt{5-2x} = 6 - x.$$

Then

$$\begin{array}{rrrrrrrr} & 6 & - & 2x & - & 2\sqrt{5-2x} & = & 6 & - & x \\ + & -6 & & 2x & & & & -6 & & 2x \\ \hline & & & & & -2\sqrt{5-2x} & = & & & x \end{array}$$

Square both sides:

$$(-2\sqrt{5-2x})^2 = x^2, \quad 4(5-2x) = x^2, \quad 20 - 8x = x^2.$$

Then

$$\begin{array}{rrrrrrrrr} & 20 & - & 8x & = & x^2 & & & \\ + & -20 & & 8x & & & & 8x & & -20 \\ \hline & 0 & & & = & x^2 & + & 8x & - & 20 \end{array}$$

Factor and solve:

$$\begin{gathered} x^2 + 8x - 20 = 0 \\ (x+10)(x-2) = 0 \end{gathered}$$

$$\swarrow \qquad\qquad \searrow$$

$$\begin{gathered} x + 10 = 0 \\ x = -10 \end{gathered} \qquad\qquad \begin{gathered} x - 2 = 0 \\ x = 2 \end{gathered}$$

Check: $x = -10$ gives

$$\sqrt{5-2x} - 1 = \sqrt{25} - 1 = 4, \quad \sqrt{6-x} = \sqrt{16} = 4 \quad \text{(Checks)}$$

$x = 2$ gives

$$\sqrt{5-2x} - 1 = \sqrt{1} - 1 = 0, \quad \sqrt{6-x} = \sqrt{4} = 2 \quad \text{(Doesn't work)}$$

The solution is $x = -10$. ☐

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14. Solve: $\dfrac{13}{x^2-4} = \dfrac{2}{x-2} - \dfrac{3}{x+2}$.

Factor the denominator on the left:

$$\frac{13}{(x-2)(x+2)} = \frac{2}{x-2} - \frac{3}{x+2}.$$

Multiply both sides by $(x-2)(x+2)$ to clear denominators:

$$(x-2)(x+2) \cdot \frac{13}{(x-2)(x+2)} = (x-2)(x+2) \cdot \left(\frac{2}{x-2} - \frac{3}{x+2}\right),$$

$$13 = 2(x+2) - 3(x-2), \quad 13 = 2x + 4 - 3x + 6, \quad 13 = 10 - x.$$

Then

$$\begin{array}{rcrcr} & 13 & = & 10 & - & x \\ - & 10 & & 10 & & \\ \hline & 3 & = & & & -x \\ \times & -1 & & & & -1 \\ \hline & -3 & = & & & x \end{array}$$

Check: $x = -3$ gives

$$\frac{13}{x^2-4} = \frac{13}{5}, \quad \frac{2}{x-2} - \frac{3}{x+2} = -\frac{2}{5} - (-3) = \frac{13}{5} \quad \text{(Checks)}$$

The solution is $x = -3$. ☐

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15. Simplify:

(a) $-(49^{3/2})$.

$$-(49^{3/2}) = -(\sqrt{49})^3 = -(7^3) = -343. \quad ☐$$

(b) $(-49)^{3/2}$.

$$(-49)^{3/2} \quad \text{is undefined.} \quad ☐$$

(c) $(-64)^{-5/3}$.

$$(-64)^{-5/3} = \frac{1}{(-64)^{5/3}} = \frac{1}{(\sqrt[3]{-64})^5} = \frac{1}{(-4)^5} = -\frac{1}{1024}. \quad ☐$$

(d) $\sqrt{360x^2y^3}$, assuming that the variables represent nonnegative quantities.

$$\sqrt{360x^2y^3} = \sqrt{360}\sqrt{x^2}\sqrt{y^3} = \sqrt{36}\sqrt{10}\sqrt{x^2}\sqrt{y^2}\sqrt{y} = 6xy\sqrt{10y}. \quad ☐$$

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16. Find values for a and b which prove that the following statement is not an algebraic identity:

$$\text{“}\sqrt{a^2+b^2} \stackrel{?}{=} a+b\text{”}$$

If $a=1$ and $b=1$, then

$$\sqrt{a^2+b^2} = \sqrt{2} \quad \text{while} \quad a+b=2.$$

Since $\sqrt{2} \neq 2$, $\sqrt{a^2+b^2} \neq a+b$ for $a=1$ and $b=1$. □

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17. (a) Combine the fractions over a common denominator: $\dfrac{1}{x^2-3x} + \dfrac{2x}{x^2-9}$.

$$\frac{1}{x^2-3x} + \frac{2x}{x^2-9} = \frac{1}{x(x-3)} + \frac{2x}{(x-3)(x+3)} = \frac{x+3}{x+3}\cdot\frac{1}{x(x-3)} + \frac{x}{x}\cdot\frac{2x}{x^2-9} =$$

$$\frac{(x+3)+2x^2}{x(x-3)(x+3)} = \frac{2x^2+x+3}{x(x-3)(x+3)}. \quad \square$$

(b) Combine the fractions over a common denominator: $\dfrac{y}{x^2-xy} - \dfrac{y}{x^2-y^2}$.

$$\frac{y}{x^2-xy} - \frac{y}{x^2-y^2} = \frac{y}{x(x-y)} - \frac{y}{(x-y)(x+y)} = \frac{y}{x(x-y)}\cdot\frac{x+y}{x+y} - \frac{y}{(x-y)(x+y)}\cdot\frac{x}{x} =$$

$$\frac{y(x+y)}{x(x-y)(x+y)} - \frac{xy}{x(x-y)(x+y)} = \frac{y(x+y)-xy}{x(x-y)(x+y)} = \frac{y^2}{x(x-y)(x+y)}. \quad \square$$

(c) Simplify: $\dfrac{\dfrac{x^2+1}{1-\dfrac{1}{x}} - \dfrac{x^2+1}{1+\dfrac{1}{x}}}{x+\dfrac{1}{x}}$.

$$\frac{\dfrac{x^2+1}{1-\dfrac{1}{x}} - \dfrac{x^2+1}{1+\dfrac{1}{x}}}{x+\dfrac{1}{x}} = \frac{\dfrac{x^2+1}{1-\dfrac{1}{x}} - \dfrac{x^2+1}{1+\dfrac{1}{x}}}{x+\dfrac{1}{x}}\cdot\frac{x}{x} = \frac{\dfrac{x(x^2+1)}{1-\dfrac{1}{x}} - \dfrac{x(x^2+1)}{1+\dfrac{1}{x}}}{x^2+1} =$$

$$\frac{x(x^2+1)}{(x^2+1)\left(1-\dfrac{1}{x}\right)} - \frac{x(x^2+1)}{(x^2+1)\left(1+\dfrac{1}{x}\right)} = \frac{x}{1-\dfrac{1}{x}} - \frac{x}{1+\dfrac{1}{x}} = \frac{x}{1-\dfrac{1}{x}}\cdot\frac{x}{x} - \frac{x}{1+\dfrac{1}{x}}\cdot\frac{x}{x} =$$

$$\frac{x^2}{x-1} - \frac{x^2}{x+1} = \frac{x^2}{x-1}\cdot\frac{x+1}{x+1} - \frac{x^2}{x+1}\cdot\frac{x-1}{x-1} = \frac{x^2(x+1)-x^2(x-1)}{(x-1)(x+1)} =$$

$$\frac{x^3+x^2-x^3+x^2}{(x-1)(x+1)} = \frac{2x^2}{(x-1)(x+1)}. \quad \square$$

---

18. Compute (without using a calculator):

(a) $\log_5 125$.

$$\log_5 125 = 3, \quad \text{because} \quad 5^3 = 125. \quad \square$$

(b) $\log_5 5^{89}$.

$$\log_5 5^{89} = 89. \quad \square$$

(c) $\log_5 \dfrac{1}{25}$.

$$\log_5 \frac{1}{25} = -2, \quad \text{because} \quad 5^{-2} = \frac{1}{25}. \quad \square$$

(d) $\log_{47} 1$.

$$\log_{47} 1 = 0. \quad \square$$

(e) $\log_8 4$.

$$\log_8 4 = \frac{2}{3}, \quad \text{because} \quad 8^{2/3} = 4. \quad \square$$

(f) $e^{\ln 42}$.

$$e^{\ln 42} = 42. \quad \square$$

---

19. Find the inverse function of $f(x) = \dfrac{x}{x-1} + 1$.

Write $y = \dfrac{x}{x-1} + 1$. Swap x's and y's: $x = \dfrac{y}{y-1} + 1$. Solve for y:

$$x = \frac{y}{y-1} + 1, \quad x - 1 = \frac{y}{y-1}, \quad (x-1)(y-1) = y, \quad (x-1)y - (x-1) = y,$$

$$-(x-1) = y - (x-1)y, \quad -(x-1) = (1 - (x-1))\,y, \quad -(x-1) = (2-x)y, \quad y = \frac{-(x-1)}{2-x}.$$

The inverse function is $f^{-1}(x) = \dfrac{-(x-1)}{2-x}$. $\square$

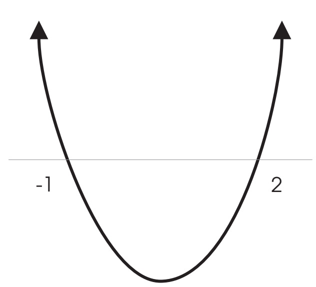
---

20. Find the domain of the function $f(x) = \dfrac{1}{\sqrt{x^2 - x - 2}}$.

$f(x) = \dfrac{1}{\sqrt{(x-2)(x+1)}}$; $x = 2$ and $x = -1$ are not in the domain, because those values cause division by zero.

I must throw out values of x for which $(x-2)(x+1) < 0$, since I can't take the square root of a negative number. To find out which values I need to throw out, I solve the inequality.

The graph of $y = (x-2)(x+1)$ is a parabola opening upward. The roots are $x = 2$ and $x = -1$.



Thus, $(x-2)(x+1) < 0$ for $-1 < x < 2$.

If I throw out these points, together with $x = 2$ and $x = -1$, I'm left with the domain: $x < -1$ or $x > 2$.

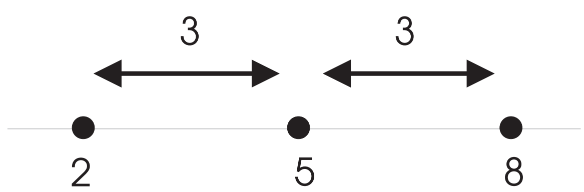
$\square$

---

21. Solve for x:

(a) $|2x - 5| = 3$.

$$\begin{array}{ccc} |2x-5| & = & 3 \\ \text{the distance from 2x to 5} & \text{is} & 3 \end{array}$$



Thus, $2x = 2$ or $2x = 8$, so $x = 1$ or $x = 4$. $\square$

(b) $3x - 5 = 2(x+1)$.

$$3x - 5 = 2(x+1), \quad 3x - 5 = 2x + 2.$$

Then

$$\begin{array}{rrrcrrr} & 3x & - & 5 & = & 2x & + & 2 \\ - & 2x & & & & 2x & & \\ \hline & x & - & 5 & = & & & 2 \\ + & & & 5 & & & & 5 \\ \hline & x & & & = & & & 7 \end{array} \quad \square$$

(c) $4(x-2) < x + 7$.

$$4(x-2) < x + 7, \quad 4(x-2) < x + 7, \quad 4x - 8 < x + 7.$$

Then

$$\begin{array}{rrrrcrrr} & 4x & - & 8 & < & x & + & 7 \\ - & x & & & & x & & \\ \hline & 3x & - & 8 & < & & & 7 \\ + & & & 8 & & & & 8 \\ \hline & 3x & & & < & & & 15 \\ / & 3 & & & & & & 3 \\ \hline & x & & & < & & & 5 \end{array} \quad \square$$

(d) $x^2 - 4x = 5$.

$$\begin{array}{rrrrrrcr} & x^2 & - & 4x & & & = & 5 \\ - & & & & & 5 & & 5 \\ \hline & x^2 & - & 4x & - & 5 & = & 0 \end{array}$$

Factor and solve:

$$x^2 - 4x - 5 = 0$$
$$(x-5)(x+1) = 0$$

$$\swarrow \qquad \qquad \searrow \qquad \square$$

$$\begin{array}{ccc} x - 5 = 0 & \qquad & x + 1 = 0 \\ x = 5 & & x = -1 \end{array}$$

(e) $x^2 - 4x = -5$.

$$\begin{array}{rrrrrrcr} & x^2 & - & 4x & & & = & -5 \\ + & & & & & 5 & & 5 \\ \hline & x^2 & - & 4x & + & 5 & = & 0 \end{array}$$

Apply the quadratic formula:

$$x = \frac{4 \pm \sqrt{16-20}}{2} = \frac{4 \pm 2i}{2} = 2 \pm i. \quad \square$$

---

22. Simplify the expressions, writing each result in the form $a + bi$:

(a) i^{33}.

$$i^{33} = i \cdot i^{32} = i \cdot (i^2)^{16} = i \cdot (-1)^{16} = i \cdot 1 = i. \quad \square$$

(b) $(3 - i)(4 + 5i)$.

$$(3-i)(4+5i) = 12 - 4i + 15i - 5i^2 = 12 + 11i - 5(-1) = 17 + 11i. \quad \square$$

(c) $\dfrac{1}{2+3i}$.

$$\frac{1}{2+3i} = \frac{1}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{2-3i}{(2+3i)(2-3i)} = \frac{2-3i}{4+9} = \frac{2}{13} - \frac{3}{13}i. \quad \square$$

(d) $\dfrac{1-2i}{6+8i}$.

$$\frac{1-2i}{6+8i} = \frac{1-2i}{6+8i} \cdot \frac{6-8i}{6-8i} = \frac{-10-20i}{100} = -\frac{1}{10} - \frac{1}{5}i. \quad \square$$

(e) $(2+3i)^2$.

$$(2+3i)^2 = 2^2 + 2(2)(3i) + (3i)^2 = 4 + 12i + 9i^2 = 4 + 12i - 9 = -5 + 12. \quad \square$$

(f) $\dfrac{3-2i}{4+i}$.

$$\frac{3-2i}{4+i} = \frac{3-2i}{4+i} \cdot \frac{4-i}{4-i} = \frac{12-3i-8i+2i^2}{16-i^2} = \frac{10-11i}{17}. \quad \square$$

---

23. Find the quotient and the remainder when $x^4 + 3x^3 - 2x^2 + 1$ is divided by $x^2 - 1$.

$$\begin{array}{r|l}
 & x^2 + 3x - 1 \\
\hline
x^2 - 1 & x^4 + 3x^3 - 2x^2 + 1 \\
 & x^4 \qquad\quad - x^2 \\
\hline
 & \quad 3x^3 - x^2 \\
 & \quad 3x^3 \qquad - 3x \\
\hline
 & \qquad\quad -x^2 + 3x + 1 \\
 & \qquad\quad -x^2 \qquad + 1 \\
\hline
 & \qquad\qquad\quad 3x
\end{array}$$

$$x^4 + 3x^3 - 2x^2 + 1 = (x^2 - 1)(x^2 + 3x - 1) + 3x. \quad \square$$

---

24. Solve the following equations, giving *exact* answers:

(a) $e^{2x} + 5 = 6e^x$.

$$\begin{array}{rrrrrrr}
 & e^{2x} & & & + & 5 & = & 6e^x \\
- & & & 6e^x & & & & 6e^x \\
\hline
 & e^{2x} & - & 6e^x & + & 5 & = & 0 \\
 & (e^x)^2 & - & 6e^x & + & 5 & = & 0
\end{array}$$

Let $y = e^x$. The equation becomes $y^2 - 6y + 5 = 0$. Factor and solve:

$$\begin{aligned} y^2 - 6y + 5 &= 0 \\ (y-1)(y-5) &= 0 \end{aligned}$$

$$\swarrow \qquad\qquad\qquad \searrow$$

$$\begin{aligned} y - 1 &= 0 \\ y &= 1 \end{aligned} \qquad\qquad\qquad \begin{aligned} y - 5 &= 0 \\ y &= 5 \end{aligned}$$

$y = 1$ gives $e^x = 1$. Then $x = \ln e^x = \ln 1 = 0$.
$y = 5$ gives $e^x = 5$. Then $x = \ln e^x = \ln 5$.
The solutions are $x = 0$ and $x = \ln 5$. □

(b) $3^{x+2} = 7^{2x+1}$.

$$\begin{array}{cccccccc} & & & 3^{x+2} & = & 7^{2x+1} & & \\ & & & \ln 3^{x+2} & = & \ln 7^{2x+1} & & \\ & & & (x+2)\ln 3 & = & (2x+1)\ln 7 & & \\ & x\ln 3 & + & 2\ln 3 & = & 2x\ln 7 & + & \ln 7 \\ - & 2x\ln 7 & & 2\ln 3 & & 2x\ln 7 & & 2\ln 3 \\ \hline & x\ln 3 - 2x\ln 7 & & & = & & & \ln 7 - 2\ln 3 \\ & x(\ln 3 - 2\ln 7) & & & = & & & \ln 7 - 2\ln 3 \\ / & \ln 3 - 2\ln 7 & & & & & & \ln 3 - 2\ln 7 \\ \hline & x & & & = & & & \dfrac{\ln 7 - 2\ln 3}{\ln 3 - 2\ln 7} \end{array}$$

The solution is $x = \dfrac{\ln 7 - 2\ln 3}{\ln 3 - 2\ln 7}$. □

(c) $4^{3x+1} = 5^x$.

$$\begin{aligned} 4^{3x+1} &= 5^x \\ \ln 4^{3x+1} &= \ln 5^x \\ (3x+1)\ln 4 &= x\ln 5 \\ (3\ln 4)x + \ln 4 &= (\ln 5)x \\ \ln 4 &= (\ln 5)x - (3\ln 4)x \\ \ln 4 &= (\ln 5 - 3\ln 4)\,x \\ \frac{\ln 4}{\ln 5 - 3\ln 4} &= x \quad \square \end{aligned}$$

(d) $\ln(x-2) + \ln(x+2) = \ln 3x$.

$$\begin{array}{cccccc} & & & \ln(x-2)(x+2) & = & \ln 3x \\ & & & e^{(\ln(x-2)(x+2))} & = & e^{(\ln 3x)} \\ & & & (x-2)(x+2) & = & 3x \\ x^2 & & - & 4 & = & 3x \\ & 3x & & & & 3x \\ \hline x^2 & - 3x & - & 4 & = & 0 \end{array}$$

Factor and solve:

$$\begin{aligned} x^2 - 3x - 4 &= 0 \\ (x-4)(x+1) &= 0 \end{aligned}$$

$$\swarrow \qquad\qquad\qquad \searrow$$

$$\begin{aligned} x - 4 &= 0 \\ x &= 4 \end{aligned} \qquad\qquad\qquad \begin{aligned} x + 1 &= 0 \\ x &= -1 \end{aligned}$$

$x = -1$ can't be plugged into the original equation, because you can't take the log of a negative number.

$x = 4$ gives

$$\ln(x-2) + \ln(x+2) = \ln 2 + \ln 6 = \ln 12 = \ln 3x.$$

The only solution is $x = 4$. $\square$

(e) $(\ln x)^2 - 3\ln x - 4 = 0$.

Let $y = \ln x$. The equation becomes

$$y^2 - 3y - 4 = 0, \quad \text{or} \quad (y-4)(y+1) = 0.$$

The solutions are $y = 4$ and $y = -1$.

$$y = 4 \quad \text{gives} \quad \ln x = 4, \quad \text{so} \quad e^{\ln x} = e^4, \quad \text{and} \quad x = e^4.$$

$$y = -1 \quad \text{gives} \quad \ln x = -1, \quad \text{so} \quad e^{\ln x} = e^{-1}, \quad \text{and} \quad x = e^{-1}.$$

The solutions are $x = e^4$ and $x = e^{-1}$. $\square$

---

25. (a) Simplify $\sqrt{500}$.

$$\sqrt{500} = \sqrt{100}\sqrt{5} = 10\sqrt{5}. \quad \square$$

(b) Simplify $\sqrt{-300}$.

$$\sqrt{-300} = \sqrt{100}\sqrt{3}\sqrt{-1} = 10i\sqrt{3}. \quad \square$$

(c) Rationalize $\dfrac{2+\sqrt{7}}{1-3\sqrt{7}}$.

$$\frac{2+\sqrt{7}}{1-3\sqrt{7}} = \frac{2+\sqrt{7}}{1-3\sqrt{7}} \cdot \frac{1+3\sqrt{7}}{1+3\sqrt{7}} = \frac{(2+\sqrt{7})(1+3\sqrt{7})}{(1-3\sqrt{7})(1+3\sqrt{7})} = \frac{2+6\sqrt{7}+\sqrt{7}+3(\sqrt{7})^2}{1-9(\sqrt{7})^2} =$$

$$\frac{2+7\sqrt{7}+3(7)}{1-9(7)} = \frac{23+7\sqrt{7}}{-62} = -\frac{23+7\sqrt{7}}{62}. \quad \square$$

---

26. Find the equation of the line:

(a) Which passes through the points $(2,3)$ and $(-11,1)$.

The slope is $\dfrac{1-3}{-11-2} = \dfrac{2}{13}$. The point-slope form for the equation of the line is

$$\frac{2}{13}(x-2) = y - 3. \quad \square$$

(b) Which passes through the point $(3,4)$ and is perpendicular to the line $2x - 8y = 5$.

$$\begin{array}{rrrrcrrr} & 2x & - & 8y & = & & & 5 \\ - & 2x & & & & 2x & & \\ \hline & & & -8y & = & -2x & + & 5 \\ / & & & -8 & & -8 & & -8 \\ \hline & & & y & = & \frac{1}{4}x & - & \frac{5}{8} \end{array}$$

The slope of the given line is $\frac{1}{4}$. The line I want is perpendicular to the given line, so the line I want has slope -4 (the negative reciprocal of $\frac{1}{4}$). The point-slope form for the equation of the line is

$$-4(x-3) = y - 4, \quad \text{or} \quad y = -4x + 16. \quad \square$$

(c) Which is parallel to the line $3y - 6x + 5 = 0$ and has y-intercept -17.

$$\begin{aligned} 3y - 6x + 5 &= 0 \\ 3y &= 6x - 5 \\ y &= 2x - \frac{5}{3} \end{aligned}$$

The given line has slope 2. The line I want is parallel to the given line, so it also has slope 2. Since it has y-intercept -17, the equation is $y = 2x - 17$. $\square$

---

27. Solve the system of equations for x and y:

$$2x + 5y = 7, \quad x + 3y = -4.$$

Multiply the second equation by 2, then subtract it from the first:

$$\begin{array}{rcrcr} 2x & + & 5y & = & 7 \\ 2x & + & 6y & = & -8 \\ \hline & & -y & = & 15 \\ & & y & = & -15 \end{array}$$

Plug this into the first equation: $2x - 75 = 7$. Then

$$\begin{array}{lcccrr} & 2x & - & 75 & = & 7 \\ + & & & 75 & & 75 \\ \hline & 2x & & & = & 82 \\ / & 2 & & & & 2 \\ \hline & x & & & = & 41 \end{array}$$

The solution is $x = 41$, $y = -15$. $\square$

---

28. Suppose $\log_a x = -5$ and $\log_a y = 2$. Find:

(a) $\log_a \dfrac{1}{\sqrt[3]{x}} = -\dfrac{1}{3} \log_a x = \dfrac{5}{3}$. $\square$

(b) $\log_a (a^6 y^2) = \log_a a^6 + 2 \log_a y = 6 + 2 \cdot 2 = 10$. $\square$

(c) $\log_a \dfrac{x^{-3}}{y^2} = (-3) \log_a x - 2 \log_a y = 15 - 4 = 11$. $\square$

---

29. (a) Simplify and write the result using positive exponents: $\dfrac{(-3x^2)^5 y^{-7}}{-9x^3 y^6}$.

$$\frac{(-3x^2)^5 y^{-7}}{-9x^3 y^6} = \frac{-243x^{10} y^{-7}}{-9x^3 y^6} = \frac{27x^7}{y^{13}}. \quad \square$$

(b) Simplify and write the result using positive exponents: $4(x^{1/3}y^{2/5})^2 \cdot (-3y^{-3/5})^2 x^{-1/6}$.

$$4(x^{1/3}y^{2/5})^2 \cdot (-3y^{-3/5})^2 x^{-1/6} = 4x^{2/3}y^{4/5} \cdot 9y^{-6/5}x^{-1/6} = 36x^{1/2}y^{-2/5} = \frac{36x^{1/2}}{y^{2/5}}. \quad \square$$

---

30. (a) Simplify, cancelling any common factors: $\dfrac{\dfrac{x^2-2x}{x^2-4}}{\dfrac{x^3-3x^2}{x^2-x-6}}$.

$$\frac{\dfrac{x^2-2x}{x^2-4}}{\dfrac{x^3-3x^2}{x^2-x-6}} = \frac{x^2-2x}{x^2-4} \cdot \frac{x^2-x-6}{x^3-3x^2} = \frac{x(x-2)}{(x-2)(x+2)} \frac{(x-3)(x+2)}{x^2(x-3)} = \frac{1}{x}. \quad \square$$

(b) Simplify, cancelling any common factors: $\dfrac{\dfrac{a^3-2a^2b}{a^3-4ab^2}}{\dfrac{a^4+3a^3b}{a^2+ab-2b^2}}$.

$$\frac{\dfrac{a^3-2a^2b}{a^3-4ab^2}}{\dfrac{a^4+3a^3b}{a^2+ab-2b^2}} = \frac{a^3-2a^2b}{a^3-4ab^2} \cdot \frac{a^2+ab-2b^2}{a^4+3a^3b} = \frac{a^2(a-2b)}{a(a-2b)(a+2b)} \cdot \frac{(a-b)(a+2b)}{a^3(a+3)} = \frac{a-b}{a^2(a+3b)}. \quad \square$$

(c) Simplify, cancelling any common factors: $\dfrac{\dfrac{x^2-5x}{x^3-4x^2}}{\dfrac{x^2-10x+25}{x^2-16}}$.

$$\frac{\dfrac{x^2-5x}{x^3-4x^2}}{\dfrac{x^2-10x+25}{x^2-16}} = \frac{x^2-5x}{x^3-4x^2} \cdot \frac{x^2-16}{x^2-10x+25} = \frac{x(x-5)}{x^2(x-4)} \cdot \frac{(x-4)(x+4)}{(x-5)^2} = \frac{x+4}{x(x-5)}. \quad \square$$

---

31. (a) Find the equation of the parabola which passes through the point $(3,7)$ and has vertex $(2,3)$.

The vertex is $(2,3)$, so the parabola's equation has the form $y = a(x-2)^2 + 3$. Plug in $x = 3$, $y = 7$:

$$7 = a(3-2)^2 + 3, \quad 7 = a + 3, \quad 7 - 3 = a + 3 - 3, \quad 4 = a.$$

The equation is $y = 4(x-2)^2 + 3$. $\square$

(b) Write the quadratic function $f(x) = x^2 - 6x + 34$ in standard (vertex) form.

Complete the square: $\left(\dfrac{-6}{2}\right)^2 = (-3)^2 = 9$, so

$$f(x) = x^2 - 6x + 34 = (x^2 - 6x + 9) + (34 - 9) = (x-3)^2 + 25. \quad \square$$

---

32. Find the center and the radius of the circle whose equation is

$$x^2 - 4x + y^2 + 6y = 12.$$

Half of -4 is -2, and $(-2)^2 = 4$. I need to add 4 to the x-stuff to complete the square.
Half of 6 is 3, and $3^2 = 9$. I need to add 9 to the y-stuff to complete the square.

$$x^2 - 4x + 4 + y^2 + 6y + 9 = 12 + 4 + 9, \quad (x-2)^2 + (y+3)^2 = 25.$$

The center is $(2, -3)$ and the radius is 5. $\square$

---

33. Suppose that \$1000 is invested at 6% annual interest, compounded monthly. How many years must pass before the account is worth at least \$2000? (Round up to the nearest year.)

The compound interest formula is

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

P is the amount invested, t is the number of years, n is the number of compoundings per year, r is the interest rate, and A is the value of the account. In this case, $P = 1000$, $n = 12$ (monthly compounding, with 12 months in a year), $r = 0.06$, and $A = 2000$. I want to find t, where

$$2000 = 1000\left(1 + \frac{0.06}{12}\right)^{12t}.$$

Simplify, and solve for t:

$$\begin{aligned} 2000 &= 1000 \cdot 1.005^{12t} \\ 2 &= 1.005^{12t} \\ \ln 2 &= \ln 1.005^{12t} \\ \ln 2 &= 12t \ln 1.005 \\ \frac{\ln 2}{12 \ln 1.005} &= t \end{aligned}$$

Thus, $t = \dfrac{\ln 2}{12 \ln 1.005} \approx 11.58131$. Rounding up, I find that the account must mature for 12 years. $\square$

The best thing for being sad is to learn something. - Merlyn, in T. H. White's *The Once and Future King*